

## ALGEBRA PROBLEM SESSION #4 SOLUTIONS

### Systems of Linear Equations

1. It is easier to use the addition method to solve a system when the coefficient of one of the variables is a multiple of the coefficient of the same variable in the other equation. It is easier to use the substitution method when one of the equations can be readily solved for one of the variables.
2. a) When using the addition method to solve a linear system, you can tell if the system has no solution if your final calculation ends with a contradiction (a false statement), e.g.  $0 = 5$ . The relationship between the two graphs is that they are parallel lines.  
b) When using the addition method to solve a linear system, you can tell if the system has infinitely many solutions if your final calculation ends with an identity (a true statement), e.g.  $2 = 2$  or  $2x = 2x$ . The relationship between the two graphs is that they coincide, i.e. they are the same line.
3. a) When using the substitution method to solve a system, you can tell if the system has no solution if your final calculation ends with a contradiction (a false statement), e.g.  $0 = 5$ . The relationship between the two graphs is that they are parallel lines.  
b) When using the substitution method to solve a system, how can you tell if the system has infinitely many solutions if your final calculation ends with an identity (a true statement), e.g.  $2 = 2$  or  $2x = 2x$ . The relationship between the two graphs is that they coincide, i.e. they are the same line.
4. You should use the substitution method since the first equation has  $y$  already solved for.
5. You should use the addition method since no variable has a coefficient of 1 or -1.
6. You can use inspection to classify a linear system as consistent and independent if neither of the following occur: the variable terms of one equation are multiples of the variable terms of the other equation but the constants terms are not multiples of each other, by the same factor as the variable terms **OR** if one equation is not a multiple of the other.

You can use inspection to classify a linear system as inconsistent if the variable terms of one equation are a multiple of the variable terms of the other equation but the constant terms are not multiples of each other, by the same factor as the variable terms.

You can use inspection to classify a linear system as consistent and dependent if you attempt to solve the system by substitution or addition, eliminate both variables, and end with an identity or if one equation is a multiple of the other.

7. A system of two **linear** equations in two variables cannot have exactly two solutions, since the system represents a pair of lines which either are parallel (no solution), intersect at one point (1 solution), or are identical (infinite number of solutions).
8. a)  $\begin{cases} y = x + 7 \\ y = -x - 3 \end{cases}$  forms an independent system of equations with a solution of  $(-5, 2)$ .  
b)  $\begin{cases} y = x + 7 \\ 2y - 2x = 14 \end{cases}$  forms a dependent system of equations with a solution of  $(-5, 2)$ .
9. a)  $(5, 1)$       b)  $(4, 1)$       c)  $y = -10$       d) Infinite Solutions  
e)  $(-\frac{1}{2}, \frac{5}{2})$       f)  $(-1, 0)$       g) No solution

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### Solving Verbal Problems Using Systems of Equations

- (a) The measures of the two angles are  $68^\circ$  and  $12^\circ$ .
- (b)  $21 \frac{1}{3}$  ounces of hamburger that is 75% lean should be mixed with  $43 \frac{1}{3}$  ounces hamburger that is 90% lean to get 65 ounces of hamburger that is 85% lean?
- (c) The speed of the wind is 50 mph and the speed of the plane in still air is 550 mph.
- (d) The island is 60 miles from shore and the traveling time to the island is 3 hours.
- (e) You have 22 20-cent coupons and 8 25-cent coupons.
- (f) The larger number is 84 and the smaller number is 56.
- (g) 15 liters of the 25% drug solution must be mixed with 35 liters of the 55% drug solution to produce 50 liters of 46% drug solution.
- (h) Amount A invested in the savings account is \$654 and amount B invested in the market account is \$446.
- (i) Given that the total of the 358 coins is \$5.95, then there are 299 pennies and 59 nickels.
- (j) There are 26 dimes and 10 quarters.