## ALGEBRA PROBLEM SESSION \#6 SOLUTIONS

## Introduction to Polynomials and Polynomial Functions

1. A polynomial function is of the form $a_{n} x^{n}+a_{n-1} x^{n-1}+a_{n-2} x^{n-2}+\cdots+a_{2} x^{2}+a_{1} x+a_{0}$ where $n$ is an integer, and each coefficient $a_{i}$ is a real number.
2. The graph of a polynomial function is smooth because it contains only rounded curves with no sharp corners and is continuous because the graph has no breaks and can be drawn without lifting your pencil from the rectangular system.
3. The leading coefficient of a polynomial is the constant or number that multiplies the highest degree term of the polynomial.
4. Like terms are terms that have the same variables raised to the same exponents. The terms $x^{2} y$ and $x y^{2}$ are not like terms, since $x^{2} y$ has the variable $x$ raised to the second power and $x y^{2}$ has the variable $x$ raised to the first power.
5. $-2 x+9$
6. $-8 x^{2}-2 x+2$
7. $7 x^{3}-x^{2}-x+3$
8. The degree of a term is found by adding the exponents of each variable. The degree of each term in the polynomial $9 x^{2} y+13 x^{2} y^{2}+8 x^{4} y^{4}$ is 3,4 , and 8 . The degree of a polynomial is equal to the degree of the term with the highest degree. In this polynomial, the term $8 x^{4} y^{4}$ has highest degree of 8 , giving the polynomial the degree of 8 .
9. Adding a polynomial of the form $a_{n} x^{n}+a_{n-1} x^{n-1}+a_{n-2} x^{n-2}+\cdots+a_{2} x^{2}+a_{1} x+a_{0}$ to another polynomial of the for $b_{n} x^{n}+b_{n-1} x^{n-1}+b_{n-2} x^{n-2}+\cdots+b_{2} x^{2}+b_{1} x+b_{0}$ gives a polynomial of the form $\mathrm{c}_{\mathrm{i}} x^{n}+c_{n-1} x^{n-1}+$ $c_{n-2} x^{n-2}+\cdots+c_{2} x^{2}+c_{1} x+c_{0}$ where each $c_{i}=a_{i}+b_{i}$ for all $i=0,1, \ldots, n$. $\left(x^{2}-2 x\right)+\left(2 x-x^{2}\right)=0$ which is a polynomial, with all $a_{i}=0$.
10. $13 m^{3}+7 m^{2}+11 m+35$
11. $-47 n^{2}+29 n-7$
12. $x^{3}+3$
13. $9 x^{2}-4 x-30$
14. Answers will vary, e.g. $\left(5 x^{3}+2 x^{2}-3 x+5\right)+\left(-2 x^{3}-4 x^{2}+4 x-12\right)=3 x^{3}-2 x^{2}+x-7$
15. Answers will vary, e.g. $\left(2 x^{3}+6 x\right)-\left(-5 x^{2}+12\right)=2 x^{3}+5 x^{2}+6 x-12$
16. $4\left(2 \mathrm{x}^{2}+3 \mathrm{x}-9\right)-7\left(3 \mathrm{x}^{2}-3 \mathrm{x}-8\right)=-13 x^{2}+33 x+20$

## Multiplication of Polynomials

1. $12 r^{2}-11 r t+2 t^{2}$
2. $-13 x^{2}+33 x+20$
3. $r^{2} s^{2}-r s t-2 t^{2}$
4. $-6 x^{4}-15 x^{3}-8 x^{2}-20 x$

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8. Usually $(\mathrm{a}+\mathrm{b})^{3}=a^{3}+3 a^{2} b+3 a b^{2}+b^{3} \neq \mathrm{a}^{3}+\mathrm{b}^{3}$, unless $a=0$ or $b=0$ or both $a=0$ and $b=$ 0 or $a=-b$. For example, $a=3$ and $b=-3$.
9. The graph of the function $f g$ can be obtained from the graphs of functions $f$ and $g$ by taking the value of $f$ and multiplying by the value of $g$ point by point.
10. $P=20 y, A=16 y^{2}$
11. $P=28 c-16, A=45 c^{2}-32 c-20$
12. $P=11 a+12, A=4 a^{2}+4 a-3$
13. $A=4 x^{2}-144$
14. $(f g)(x)=x^{3}+27 ;(f g)(-2)=19 ;(f g)(0)=27$
15. $(f g)(x)=x^{2}+6 x-40 ;(f g)(-1)=-45 ;(f g)(0)=-40$

## Finding the Greatest Common Factor and Factoring by Grouping

1. To find the greatest common factor (gcf) of two natural numbers, list all the factors of each natural number and the gcf will be the greatest number (factor) that belongs to both lists.
2. $a c-a d+b d-b c=(a-b)(c-d)$. Yes, the results the same. There will always be one and only one factorization.
3. True
4. 

(a) $7 y\left(6 x^{2} y-4 x z-3 y z^{2}\right)$
(b) $-2 v\left(3 u^{2}-2 u+4 v\right)$
(c) $\quad-5 m n(m n-7)$
(d) $8 x^{2} y^{2}(6 x y-8 y+9)$
5. (a) $(x+1)(3 x+24)$
(b) $(r-2)(r-1)$
(c) $\quad(2-x)(5 x+4)$
(d) $2(3 y-5)(2 y-4)$
(e) $\quad(2 x-3)(5 x+6)$

## Factoring Trinomials

1. Explain what clues one should look for to assist in factoring:
(a) When factoring a polynomial of two terms you should look for common factors between the terms, a difference of squares and/or a sum or difference of cubes.
(b) When factoring a polynomial of three terms you should look for patterns similar to the special products: $x^{2}+2 x y+y^{2}=(x+y)^{2} ; x^{2}-2 x y+y^{2}=(x-y)^{2}$
2. (a) $2(x-5)(x+3)$
(b) $(2 x+1)(5 x-7)$
(c) $\quad-3(2 x+5)(4 x-7)$
3. True
4. Yes, $x^{6}-7 x^{3}+10=\left(x^{3}-5\right)\left(x^{3}-2\right)$
5. (a) $(2 x+5)(3 x+4)$
(b) $(2 x-3)(5 x-1)$
(c) $(y-7)(3 y+2)$
(d) $5(r+4)(r+13)$
(e) prime
(f) prime
6. $7 r(x-1)\left(x^{2}-3 x-3\right)$
7. $3 m(m-1)(3 m-7)$

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8. $9 n(n-2)(n+5)$ or $9\left(n^{3}+3 n^{2}-10 n\right)$
