ALGEBRA PROBLEM SESSION #7 SOLUTIONS

Factoring Special Forms

1. Perfect square trinomials are of the form: $(x+y)^2 = x^2 + 2xy + y^2$ or $(x-y)^2 = x^2 - 2xy + y^2$. The last term is obtained by squaring the second term in the binomial and the square of a term with real coefficients is positive.

2. Factor the polynomial as the difference of two squares first, since the factors will then be the sum and difference of two cubes, eg. $a^6 - b^6 = (a^3)^2 - (b^3)^2 = (a^3 + b^3)(a^3 - b^3) = (a+b)(a^2 - ab + b^2)(a-b)(a^2 + ab + b^2)$

3. Since $x^2 - 4x + 4 = (x-2)(x-2)$, the equation $x^2 - 4x + 4 = 0$, has 2 solutions of $x = 2$ or a double root at 2 or 1 root of 2 with multiplicity of 2.

4. a) $(4x-3)(4x+3)$
   b) $(3y-4)(3y+4)(9y^2+16)$
   c) $(4x-5y)^2$
   d) $x^2 - 9xy + 81y^2$
   e) $(x-y-6)(x+y-6)$
   f) $-(x^2 - x - 8)(x^2 + x + 8)$
   g) $(3y+1)(9y^2 - 3y + 1)$
   h) $(xy+4)(x^2y^2 - 4xy + 16)$
   i) $(5x^2 - y^2)(25x^4 + 5x^2y^2 + y^4)$

5. $36x^2 - 16$ square units

6. If $x = y$ then division by the factor $(x-y) = 0$ is undefined.

General Factoring

1. a) $3x^2(x^2+9)$
   b) $2x^6 + x^3 - 32$
   c) $5y(y-7)(y-2)$
   d) $6(y^2 - 11)$
   e) $y(3y+1)(9y^2 - 3y + 1)$
   f) $(x^2 - x + 1)(x^2 + x + 1)$

2. a) True b) False

3. $3x(x+y) = 3x^2 + 3xy$

Applications of Polynomial Equations

1. Find a quadratic equation with the given roots.
   (a) $x^2 - 8x + 15 = 0$ (b) $x^2 - 4x - 12 = 0$ (c) $x^2 - \frac{5}{6}x + \frac{1}{6} = 0$

2. (a) $n = \frac{1}{2}$ or $n = 1$
   (b) $r = -3$ or $r = 1$
   (c) $y = 1$
   (d) $x = -5$ or $x = 3$
   (e) $x = -3$ or $x = -\frac{5}{2}$
   (f) $x = -6$ or $x = 3$
   (g) $x = -1$ or $x = 1$ or $x = 2$

3. $c = \frac{1}{5}$ or $c = 2$

4. The length of the rectangular park 15 yards and the width is 12 yards.

5. The number is either –4 or 8.

6. The length of the original square is 4 inches.