ALGEBRA PROBLEM SESSION #10 - PRACTICE PROBLEMS

Rational Exponents

- 1. Explain how you would decided whether $a^{\frac{1}{n}}$ is a real number.
- The fraction $\frac{2}{4}$ is equal to $\frac{1}{2}$. Is $16^{\frac{2}{4}}$ equals to $16^{\frac{1}{2}}$? Explain. 2.
- 3. How would you evaluate an expression with a mixed-number exponent? For example, what is $8^{\frac{1}{3}}$? Discuss.
- Can you find any numbers a and b such that $\sqrt{a+b} = \sqrt{a} + \sqrt{b}$? 4.
- Simplify each example below. Assume that all variables represent positive numbers and write all answers without 5. negative exponents.

(a)
$$\left(a^{\frac{2}{3}}+b^{\frac{2}{3}}\right)\left(a^{\frac{2}{3}}-b^{\frac{2}{3}}\right)$$
 (b) $(81x^{4}y^{2})^{1/4}$

6. Write each expression in radical notation and simplify if possible

(a)
$$(-64)^{\frac{1}{3}}$$
 (b) $(3xy^4)^{\frac{1}{5}}$

- 7. Write each expression with rational exponents
 - (b) $\left(\sqrt{11xy}\right)^3$ (a) $\sqrt[5]{13x}$
- 8. Write each expression with a positive rational exponent and simplify if possible
 - (b) $(4xy)^{-\frac{4}{7}}$ (a) $81^{-\frac{5}{4}}$
- 9. Simplify:

(a)
$$\frac{x^{\frac{3}{7}}}{x^{\frac{1}{7}}}$$
 (b) $\left(y^{-\frac{3}{4}}\right)^{\frac{1}{6}}$ (c) $\left(\sqrt[3]{xy}\right)^{21}$ (d) $\sqrt[4]{x^2y^6}$ (e) $\frac{\sqrt[3]{x}}{\sqrt[4]{x}}$

10. How can you tell if an expression with rational exponents is simplified?

Multiplying and Simplifying Radical Expressions

1. Simplify each example below. Assume that all variables represent positive numbers and write all answers without negative exponents. 8

$$2x\sqrt{8} + 2\sqrt{200x^2} + \sqrt{50x^2} \qquad \qquad \sqrt[3]{54} - 3\sqrt[3]{16} + 4\sqrt[3]{128}$$

Multiply and simplify: 2.

a.
$$\sqrt{x+6} \cdot \sqrt{x-6}$$
 b. $\sqrt{\frac{2x}{5}} \cdot \sqrt{\frac{5}{2}}$ c. $\sqrt{2x^7} \cdot \sqrt{12x^4}$ d. $\sqrt[5]{8x^4y^3z^3} \cdot \sqrt[5]{8xy^9z^8}$

3. Simplify by factoring: a. $\sqrt{28}$ b. $\sqrt[3]{-32x^2y^3}$ c. $\sqrt[3]{x^3y^{17}z^2}$ d. $\sqrt[5]{y^{18}}$

Adding, Subtracting, and Dividing Radical Expressions

1. Simplify each example below. Assume that all variables represent positive numbers and write all answers without negative exponents.

$$2\sqrt[4]{32x^5} + 4\sqrt[4]{162x^5} - 5x\sqrt[4]{512x} \qquad \qquad 2x\sqrt{8} + 2\sqrt{200x^2} + \sqrt{50x^2}$$

- 2. A "Don't Block the Box" sign in an intersection in a certain city consists of an X pained from corner to corner within a square. Each of the lines that make up the X is 30 feet long. How wide are the streets?
- 3. Add or subtract and simplify

a.
$$9\sqrt[3]{7} - 4\sqrt[3]{7}$$

b. $8\sqrt{17} - 5\sqrt{19} - 6\sqrt{17} + 4\sqrt{19}$
c. $\sqrt[3]{54xy^3} + y\sqrt[3]{128x}$
d. $\sqrt{4x - 12} + \sqrt{x - 3}$

4. Simplify using the quotient rule:

a.
$$\sqrt[3]{\frac{x^5}{125y^3}}$$
 b. $\sqrt[5]{\frac{64x^{14}}{y^{15}}}$

- 5. Divide and simplify:
 - a. $\frac{\sqrt[3]{54}}{\sqrt[3]{2}}$ b. $\frac{\sqrt{500x^3}}{\sqrt{10x^{-1}}}$ c. $\frac{\sqrt{50xy}}{2\sqrt{2}}$
- 6. Find the perimeter and area of the rectangle. Express the answer in simplified radical form.



Multiplying and Rationalizing Denominator

1. Simplify as indicated:
$$\frac{4\sqrt{72} + 16\sqrt{36} - 2\sqrt{144}}{\sqrt{36}}$$

2. What factor is necessary to rationalize the denominator of $\frac{1}{5\sqrt{2}}$?

Multiply and simplify:
3.
$$\sqrt{6}(4\sqrt{6} - 3\sqrt{2})$$
4. $(4 + \sqrt{5})(10 - 3\sqrt{5})$
5. $(4\sqrt{3} + 3\sqrt{2})(4\sqrt{3} - 3\sqrt{2})$

Rationalize the denominator and simplify:

6.
$$\frac{10}{\sqrt[3]{4x^2}}$$
 7. $-\sqrt{\frac{150a^3}{b^5}}$ 8. $\frac{15}{\sqrt{6}+1}$ 9. $\frac{5\sqrt{3}-3\sqrt{2}}{3\sqrt{2}-2\sqrt{3}}$ 10. $\frac{3\sqrt{x}+\sqrt{y}}{\sqrt{y}-3\sqrt{x}}$

Selected problems were taken from Blitzer Algebra For College Students