## ALGEBRA PROBLEM SESSION \#10 - PRACTICE PROBLEMS

## Rational Exponents

1. Explain how you would decided whether $\mathrm{a}^{\frac{1}{n}}$ is a real number.
2. The fraction $\frac{2}{4}$ is equal to $\frac{1}{2}$. Is $16^{\frac{2}{4}}$ equals to $16^{\frac{1}{2}}$ ? Explain.
3. How would you evaluate an expression with a mixed-number exponent? For example, what is $8^{1 \frac{1}{3}}$ ? Discuss.
4. Can you find any numbers a and b such that $\sqrt{a+b}=\sqrt{a}+\sqrt{b}$ ?
5. Simplify each example below. Assume that all variables represent positive numbers and write all answers without negative exponents.
(a) $\quad\left(a^{\frac{2}{3}}+b^{\frac{2}{3}}\right)\left(a^{\frac{2}{3}}-b^{\frac{2}{3}}\right)$
(b) $\quad\left(81 x^{4} y^{2}\right)^{1 / 4}$
6. Write each expression in radical notation and simplify if possible
(a) $(-64)^{\frac{1}{3}}$
(b) $\left(3 x y^{4}\right)^{\frac{1}{5}}$
7. Write each expression with rational exponents
(a) $\sqrt[5]{13 x}$
(b) $(\sqrt{11 x y})^{3}$
8. Write each expression with a positive rational exponent and simplify if possible
(a) $81^{-\frac{5}{4}}$
(b) $(4 x y)^{-\frac{4}{7}}$
9. Simplify:
(a) $\frac{x^{\frac{3}{7}}}{x^{\frac{1}{7}}}$
(b) $\left(y^{-\frac{3}{4}}\right)^{\frac{1}{6}}$
(c) $(\sqrt[3]{x y})^{21}$
(d) $\sqrt[4]{x^{2} y^{6}}$
(e) $\frac{\sqrt[3]{x}}{\sqrt[4]{x}}$
10. How can you tell if an expression with rational exponents is simplified?

## Multiplying and Simplifying Radical Expressions

1. Simplify each example below. Assume that all variables represent positive numbers and write all answers without negative exponents.

$$
2 x \sqrt{8}+2 \sqrt{200 x^{2}}+\sqrt{50 x^{2}} \quad \sqrt[3]{54}-3 \sqrt[3]{16}+4 \sqrt[3]{128}
$$

2. Multiply and simplify:
a. $\sqrt{x+6} \cdot \sqrt{x-6}$
b. $\sqrt{\frac{2 x}{5}} \cdot \sqrt{\frac{5}{2}}$
c. $\sqrt{2 x^{7}} \cdot \sqrt{12 x^{4}}$
d. $\sqrt[5]{8 x^{4} y^{3} z^{3}} \cdot \sqrt[5]{8 x y^{9} z^{8}}$
3. Simplify by factoring:
a. $\sqrt{28}$
b. $\sqrt[3]{-32 x^{2} y^{3}}$
c. $\sqrt[3]{x^{3} y^{17} z^{2}}$
d. $\sqrt[5]{y^{18}}$
4. Simplify each example below. Assume that all variables represent positive numbers and write all answers without negative exponents.
$2 \sqrt[4]{32 x^{5}}+4 \sqrt[4]{162 x^{5}}-5 x \sqrt[4]{512 x}$

$$
2 x \sqrt{8}+2 \sqrt{200 x^{2}}+\sqrt{50 x^{2}}
$$

2. A "Don't Block the Box" sign in an intersection in a certain city consists of an $X$ pained from corner to corner within a square. Each of the lines that make up the X is 30 feet long. How wide are the streets?
3. Add or subtract and simplify
a. $\quad 9 \sqrt[3]{7}-4 \sqrt[3]{7}$
b. $8 \sqrt{17}-5 \sqrt{19}-6 \sqrt{17}+4 \sqrt{19}$
c. $\sqrt[3]{54 x y^{3}}+y \sqrt[3]{128 x}$
d. $\sqrt{4 x-12}+\sqrt{x-3}$
4. Simplify using the quotient rule:
a. $\sqrt[3]{\frac{x^{5}}{125 y^{3}}}$
b. $\sqrt[5]{\frac{64 x^{14}}{y^{15}}}$
5. Divide and simplify:
a. $\frac{\sqrt[3]{54}}{\sqrt[3]{2}}$
b. $\frac{\sqrt{500 x^{3}}}{\sqrt{10 x^{-1}}}$
c. $\frac{\sqrt{50 x y}}{2 \sqrt{2}}$
6. Find the perimeter and area of the rectangle. Express the answer in simplified radical form.


## Multiplying and Rationalizing Denominator

1. Simplify as indicated: $\frac{4 \sqrt{72}+16 \sqrt{36}-2 \sqrt{144}}{\sqrt{36}}$
2. What factor is necessary to rationalize the denominator of $\frac{1}{\sqrt[5]{2}}$ ?

Multiply and simplify:
3. $\sqrt{6}(4 \sqrt{6}-3 \sqrt{2})$
4. $(4+\sqrt{5})(10-3 \sqrt{5})$
5. $(4 \sqrt{3}+3 \sqrt{2})(4 \sqrt{3}-3 \sqrt{2})$

Rationalize the denominator and simplify:
6. $\frac{10}{\sqrt[3]{4 x^{2}}}$
7. $-\sqrt{\frac{150 a^{3}}{b^{5}}}$
8. $\frac{15}{\sqrt{6}+1}$
9. $\frac{5 \sqrt{3}-3 \sqrt{2}}{3 \sqrt{2}-2 \sqrt{3}}$
10. $\frac{3 \sqrt{x}+\sqrt{y}}{\sqrt{y}-3 \sqrt{x}}$

