## ALGEBRA PROBLEM SESSION \#13 - PRACTICE PROBLEMS

## Equations that are Quadratic in Form

1. Let $f(x)=|x|$.
a. Graph $y=f(x)+k$ for $k=-2,0$, and 1 simultaneously in the coordinate system. Describe the relationship between the graph of $y=f(x)$ and the graph of $y=f(x)+k$ for $k$, any real number.
b. Graph $y=f(x+h)$ for $h=-2,0$, and 1 simultaneously in the same coordinate system. Describe the relationship between the graph of $y=f(x)$ and the graph of $y=f(x+h)$ for $h$, any real number.
2. Given $f(x)=|x|$, which of the following represent graphs that are reflections about the $x$-axis:
a) $y=-|x|$
b) $y=|-x|$
3. Is the graph of $f(x)=\sqrt{-x}$ a reflection of $f(x)=\sqrt{x}$ about the $x$-axis or $y$-axis?
4. Graph the piecewise-defined function below, state its domain and range:

$$
f(x)=\left\{\begin{array}{c}
-x \text { if } x>0 \\
x^{2} \quad \text { if }-1 \leq x<0 \\
1 \text { if } x<-1
\end{array}\right.
$$

5. What function is shown in the example to the right:

6. A cab company charges $\$ 2$ for a trip up to 1 mile, and $\$ 1.50$ for every extra mile (or portion of a mile). Draw a graph representing the cost $(c)$ of a trip as a function of miles $(m)$ traveled.
7. Find a piecewise-defined function that is constant on the interval ( $-\infty,-1$ ), increasing on the interval $(-1,5)$, undefined at $x=5$, and decreasing on the interval $(5, \infty)$.
8. Evaluate each piecewise function at the given values of independent variable.
a) $f(x)=\left\{\begin{array}{ll}6 x-1 & \text { if } x<0 \\ 7 x+3 & \text { if } x \geq 0\end{array} \quad f(-3), f(0), f(4)\right.$
b) $g(x)=\left\{\begin{array}{ll}x+5 & \text { if } x \geq-5 \\ -(x+5) & \text { if } x<-5\end{array} \quad g(0), g(-6), g(-5)\right.$
9. The domain of each piecewise function is $(-\infty, \infty)$. Graph each function. Use the graph to determine the function's range.
a) $f(x)= \begin{cases}x & \text { if } x<0 \\ -x & \text { if } x \geq 0\end{cases}$
b) $f(x)= \begin{cases}x+2 & \text { if } x<-3 \\ x-2 & \text { if } x \geq-3\end{cases}$
10. Find and simplify the difference quotient $\frac{f(x+h)-f(x)}{h}, h \neq 0$ for the given functions.
a) $f(x)=7 x$
b) $f(x)=x^{2}-5 x+8$
c) $f(x)=-3 x^{2}+2 x-1$
d) $f(x)=\frac{1}{2 x}$
11. First graph the standard quadratic function, $f(x)=x^{2}$. Then use transformations of this graph to graph the given functions.
a) $g(x)=x^{2}-1$
b) $h(x)=-(x-1)^{2}$
c) $h(x)=-2(x+2)^{2}+1$
12. First graph the square root function, $f(x)=\sqrt{x}$. Then use transformations of this graph to graph the given functions.
a) $h(x)=-\sqrt{x+1}$
b) $g(x)=2 \sqrt{x+1}$
13. First graph the absolute value function, $f(x)=|x|$. Then use transformations of this graph to graph the given functions.
a) $g(x)=|x|+3$
b) $h(x)=|x+3|-2$
c) $g(x)=-|x+4|+2$
14. Solve each equation by making an appropriate substitution. If at any point in the solution process both sides of an equation are raised to an even power, a check is required.
a) $x^{4}-9 x^{2}+20=0$
b) $x-6 x^{\frac{1}{2}}+8=0$
c) $\left(x^{2}-2\right)^{2}-\left(x^{2}-2\right)=6$
d) $20 x^{-2}+9 x^{-1}+1=0$
e) $x^{\frac{2}{3}}+2 x^{\frac{1}{3}}-3=0$

## Exponential Functions

1. (a) In the definition of the exponential function, b could not equal 0 . Why not?
(b) In the definition of the exponential function, b could not be negative. Why not?
2. Graph the function defined by $y=\left(\frac{2}{3}\right)^{x}$
3. Graph each function by making a table of coordinates.
(a) $f(x)=5^{x}$
(b) $h(x)=\left(\frac{1}{3}\right)^{x}$
4. Graph functions $f$ and $g$ is the same rectangular coordinate system. Select integers from -2 to 2 , inclusive, for $x$. Then describe how the graph of $g$ is related to the graph of $f$.
a. $f(x)=2^{x}$ and $g(x)=2^{x+2}$
b. $f(x)=2^{x}$ and $g(x)=2^{x}+2$
c. $f(x)=2^{x}$ and $g(x)=2^{x}-1$
d. $f(x)=3^{x}$ and $g(x)=3^{-x}$
5. Find the domain and range:

6. According to the linear model, how many millions of words were in the federal tax code in 1975? According to the exponential model, how many millions of words were in the federal tax code in 1975 ? Which function is a better model for the data in 1975 ?
7. What is an exponential function?
8. What is the natural exponential function?
9. Determine if the following statement makes sense or does not make
 sense and explain your reasoning: I'm using a photocopier to reduce an image over and over by $50 \%$, so the exponential function $f(x)=\left(\frac{1}{2}\right)^{x}$ models the new image size, where $x$ is the number of reductions.

## Composite and Inverse Functions

1. Explain how to find the domain of $f / \mathrm{g}$.
2. Is composition of functions associative? Choose functions $f, \mathrm{~g}$, and h and determine whether $[f \circ(\mathrm{~g} \circ \mathrm{~h})](\mathrm{x})=[(f \circ \mathrm{~g}) \circ \mathrm{h}](\mathrm{x})$.
3. Choose functions $f, \mathrm{~g}$ and h and determine whether $\mathrm{f} \circ(\mathrm{g}+\mathrm{h})=f \circ \mathrm{~g}+\mathrm{f} \circ \mathrm{h}$.
4. Let $f(x)=x^{2}+3$ and $g(x)=2 x-7$. Find each.
a. $(g \circ f)(-3)$
b. $(f \circ g)(3)$
c. $(\mathrm{f}+\mathrm{g})(2)$
d. $(f-g)(-2)$
e. $(f \cdot g)(-1)$
f. $(g \circ f)(x)$
g. $(\mathrm{g} \circ \mathrm{g})(\mathrm{x})$
5. If $f(x)=2 x^{2}-4 x+3$ find
a. $\mathrm{f}(\mathrm{a})$
b. f(h)
c. $f(a+h)$
d. $\mathrm{f}(\mathrm{a})+\mathrm{f}(\mathrm{h})$
6. Let $f(x)=2 x^{2}-3$ and $g(x)=x^{2}-4$. Find the domain and function of $f-g$ and $g \cdot f$.
7. If $f(x)=x^{2}+3 x-4$, find
a. $f(x)=3 x+4$
b. $f(x)=x^{2}-1$
c. $f(x) x^{2}-4 x+3$
8. If $\mathrm{f}(\mathrm{x})=2 \mathrm{x}^{3}+3$ find $\frac{f(x+h)-f(x)}{h}$
9. Find $\boldsymbol{f}(\boldsymbol{g}(\boldsymbol{x}))$ and $\boldsymbol{g}(\boldsymbol{f}(\boldsymbol{x}))$ and determine whether each pair of functions $f$ and $g$ are inverses of each other.
a. $\boldsymbol{f}(\boldsymbol{x})=6 \boldsymbol{x}$ and $\boldsymbol{g}(\boldsymbol{x})=\frac{\boldsymbol{x}}{6}$
b. $f(x)=3 x-7$ and $g(x)=\frac{x+3}{7}$
c. $f(x)=\frac{2}{x-5}$ and $g(x)=\frac{2}{x}+5$
10. The following functions are all one-to-one. For each function (1) find an equation for $\boldsymbol{f}^{-1}(\boldsymbol{x})$, the inverse function and (2)verify that your equation is correct by showing that $f\left(f^{-\mathbf{1}}(x)\right)=x$ and $f^{-1}(f(x))=x$
a. $\boldsymbol{f}(\boldsymbol{x})=\boldsymbol{x}+\mathbf{5}$
b. $f(x)=x^{3}-1$
c. $f(x)=\frac{2 x-3}{x+1}$
11. Does this graph represent a function that has an inverse function?

12. Use this graph to draw the graph of its inverse function.

13. Explain how to determine if two functions are inverses of each other.
14. Describe how to find the inverse of a one-to-one function.
15. Describe how to use the graph of a one-to-one function to draw the graph of its inverse function.
