

# Solving Systems of Equations Worksheet

Solve the following systems for the unknown variables. Some might have no solution or have infinite solutions.

1.  $x - 2y = -6$   
 $y - 3x = 3$

2.  $5m = n - 10$   
 $5m = n + 10$

3.  $-2h = -3k + 5$   
 $-4h = -6k + 10$

## Economics

In economics, a market is said to be in equilibrium when quantity demanded (denoted  $Q_D$ ) is equal to quantity supplied (denoted  $Q_S$ ). Also suppose that quantity is in thousands of units (so  $Q = 3.2$  means a quantity of 3200 goods). The price at which the market is equilibrium is called the equilibrium or market clearing price. Use this fact and your knowledge of systems of equations to solve question 4 and 5.

- Suppose that demand equation is  $P = 9 - 4Q_D$  and the supply equation is  $Q_S = 11 - 9P$ . What is the equilibrium price and quantity?
- Suppose that demand equation is  $P = 9 - 4Q_D$  and the supply equation is  $Q_S = 11 - 9P$ . Suppose the market price is \$3. Is the market in equilibrium?
- A certain amount of money was deposited at an 8% annual interest rate and a second amount was deposited at a 5% annual interest rate. The total amount of money deposited for one year was \$1500 and the interest rate accumulated from the two investments was \$99. Find how much was deposited at each rate.

## Education

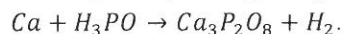
- A student has two test scores. The difference between the two scores is 12 and the mean of the scores is 80. What are the two test scores?
- For every exam taken, a student receives 3 points for a correct answer and has two points deducted for every wrong answer. If there are 40 questions on the test and the student received a grade of 85, how many questions did she get correct and how many were incorrect?
- For purposes of comparison, a teacher wants to rescale the scores on a difficult set of papers so that the maximum possible is still 100 but the mean (average) is 80 instead of 56. Write a linear formula which would do this and then decide if 60 is the lowest passing mark, what was the lowest passing mark on the original scale?

## General

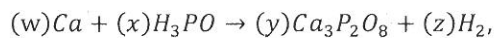
- A school is going on a fieldtrip. For every 8 students present there has to be one chaperone. There are 81 people on the fieldtrip. How many are chaperones and how many are students?
- A caterer finds that the expenses for a breakfast consist of two parts. The first part is fixed costs. The second is cost per guest. If the expense for 40 guests is \$150 and for 100 guests is \$225, find the fixed costs and the cost per guest.

## Chemistry

- Chemical equations can be balanced as in the following example. Beginning with the unbalanced equation



The problem is to determine numbers of molecules of each of the four chemicals so that the equation will be balanced. We want



where  $w, x, y, z$  are numbers of molecules of the respective compounds. Equating the number of atoms of each elements gives the following: Calcium:  $w = 3y$ , Hydrogen:  $3x = 2z$ , Phosphorus:  $x = 2y$ , Oxygen:  $4x = 8y$ . Solve for  $w, x, y$  and  $z$ .

# Systems of Equations

$$(1) \quad x - 2y = -6$$

$$y - 3x = 3$$

First, solve for  $y \rightarrow y = 3 + 3x \rightarrow$  plug  $y$  into 1st equation

$$x - 2(3 + 3x) = -6$$

$$x - 6 - 6x = -6$$

$$\begin{array}{r} x - 6 - 6x = -6 \\ +6 \qquad \qquad +6 \\ \hline \end{array}$$

$$\begin{array}{r} x - 6x = 0 \\ -5x = 0 \\ \hline -5 \quad -5 \end{array}$$

Check:

$$x - 2y = -6$$

$$0 - 2(3) = -6$$

$$0 - 6 = -6$$

$$-6 = -6 \checkmark$$



$$y - 3x = 3$$

$$3 - 3(0) = 3$$

$$3 - 0 = 3$$

$$3 = 3 \checkmark$$

$$\boxed{x = 0} \rightarrow \text{plug back into 2nd eqn to find } y$$

$$y - 3(0) = 3$$

$$y - 0 = 3$$

$$\boxed{y = 3}$$

$$(2) \quad 5m = n - 10$$

$$5m = n + 10$$

Solve  $5m = n - 10$  for  $n$

$$n = 5m + 10$$

Sub above into  $5m = n + 10$

$$5m = (5m + 10) + 10$$

$$5m = 5m + 10 + 10$$

$$5m = 5m + 20$$

$$\begin{array}{r} 5m = 5m + 20 \\ -5m \quad -5m \\ \hline \end{array}$$

$$0 = 20 \quad X$$

Solve  $5m = n + 10$  for  $n$

$$5m = n + 10$$

$$-10 \quad -10$$

$$\hline 5m - 10 = n$$

Plug into 1st equation

$$5m = n - 10$$

$$5m = (5m - 10) - 10$$

$$5m = 5m - 10 - 10$$

$$5m = 5m - 20$$

$$\begin{array}{r} 5m = 5m - 20 \\ -5m \quad -5m \\ \hline \end{array}$$

$$0 = -20 \quad X$$

no solution!

$$\textcircled{3} \quad -2h = -3K + 5$$

$$-4h = -6K + 10$$

Solve  $\frac{-4h}{-4} = \frac{-6K + 10}{-4}$  for h

$$h = \frac{-6K + 10}{-4}$$

$$h = \frac{6}{4}K + \frac{10}{-4}$$

$$h = \frac{3}{2}K - \frac{5}{2}$$

Plug h into 1st eqn:

$$-2h = -3K + 5$$

$$-2\left(\frac{3}{2}K - \frac{5}{2}\right) = -3K + 5$$

$$-\frac{6}{2}K + \frac{10}{2} = -3K + 5$$

$$\begin{array}{r} -3K + 5 = -3K + 5 \\ +3K \quad \quad \quad +3K \\ \hline 5 = 5 \checkmark \end{array}$$

$$5 = 5 \checkmark$$

Solve  $\frac{-2h}{-2} = \frac{-3K + 5}{-2}$  for h

$$h = \frac{3}{2}K - \frac{5}{2}$$

Plug into 2nd eqn:

$$-4\left(\frac{3}{2}K - \frac{5}{2}\right) = -6K + 10$$

$$-\frac{12}{2}K + \frac{20}{2} = -6K + 10$$

$$\begin{array}{r} -6K + 10 = -6K + 10 \\ +6K \quad \quad \quad +6K \\ \hline 10 = 10 \end{array}$$

$$10 = 10$$

Thus, there are infinitely many solutions to this system

$$\textcircled{4} \quad P = 9 - 4Q_D, \quad Q_S = 11 - 9P$$

Find P and  $Q_D$ .

$$P = 9 - 4Q_D \rightarrow \text{Solve for } Q_D$$

$$\begin{array}{r} P - 9 = -4Q_D \\ \hline -4 \end{array}$$

$$\frac{P - 9}{-4} = \frac{-4Q_D}{-4}$$

$$\frac{-P + 9}{4} = Q_D$$

Set  $Q_D$  and  $Q_S$  equal to each other to find equilibrium

$$Q_D = 4\left(\frac{-P + 9}{4}\right) = (11 - 9P) = Q_S$$

$$\begin{array}{r} -P + 9 = 11 - 9P \\ -9 \quad \quad -9 \\ \hline -P = 2 - 9P \end{array}$$

$$-P = 2 - 9P$$

$$\begin{array}{r} -P = 2 - 9P \\ +36P \quad \quad +36P \\ \hline 35P = 2 \\ \frac{35P}{35} = \frac{2}{35} \end{array}$$

$$P = \frac{2}{35} \rightarrow \text{equilibrium price} = \frac{2}{35}$$

$\textcircled{2}$

Now plug  $P$  into  $Q_s$  and  $Q_D$  to find the quantities

$$P = \$1.00$$

$$Q_s = 11 - 9P \longrightarrow Q_s = 11 - 9(P)$$

$$P = 9 - 4Q_D$$

$$= 11 - 9(1)$$

$$= 11 - \underline{9}$$

$$Q_s = 2$$



$$P = 9 - 4Q_D$$

$$1 = 9 - 4Q_D$$

$$\underline{-9 \quad -9}$$

$$\underline{-8} = \underline{-4} Q_D$$

$$\underline{-4} \quad \underline{-4}$$

$$Q_D = 2$$

Therefore, the equilibrium price,  $P$ ,

is \$1.00,

The Quantity supplied,  $Q_s$ , is 2,

and The Quantity demand,  $Q_D$ ,

is 2.

$$\textcircled{5} P = 9 - 4Q_D \quad P = \text{\$}3$$

$$Q_S = 11 - 9P$$

$$3 = 9 - 4Q_D$$

$$\begin{array}{r} -9 \\ -9 \end{array}$$

$$\underline{-6 = -4Q_D}$$

$$\begin{array}{r} -4 \\ -4 \end{array}$$

$$\frac{3}{2} = Q_D$$

$$Q_S = 11 - 9(3)$$

$$= 11 - 27$$

$$Q_S = -16$$

$Q_D \neq Q_S \rightarrow$  No MARKET is not in equilibrium

$$\textcircled{6} \left. \begin{array}{l} x = \text{amount invested at } 8\% \\ y = \text{amount invested at } 5\% \end{array} \right\} x + y = 1500 \text{ (TOTAL)}$$

$$\frac{.08x}{x} + \frac{(1500 - x) \cdot .05}{y} = .99$$

$$.08x + 75 - .05x = .99$$

$$.03x + 75 = .99$$

$$\begin{array}{r} -75 \\ -75 \end{array}$$

$$\frac{.93x}{.03} = \frac{24}{.03} \quad x = 800$$

$$x + y = 1500$$

Plug in  $x = 800$ , solve for  $y$

$$(800) + y = 1500$$

$$\begin{array}{r} -800 \\ -800 \end{array}$$

$$y = 700$$

$\text{\$}800$  invested at  $8\%$ .  
 $\text{\$}700$  invested at  $5\%$ .

$$\textcircled{7} \text{ Let } x = \text{Test 1} \rightarrow 86$$

$$x - 12: \text{Test 2} \rightarrow 86 - 12 = 74$$

$$\frac{x + (x - 12)}{2} = 80$$

$$\frac{x + x - 12}{2} = 80$$

$$x \left( \frac{2x - 12}{2} \right) = 80(2)$$

$$\begin{array}{r} 2x - 12 = 160 \\ +12 \quad +12 \\ \hline \end{array}$$

$$\frac{2x}{2} = \frac{172}{2}$$

$$x = 86$$

Scores:

86 and 74

8) Let  $x \rightarrow$  correct (3 pts)

$40-x \rightarrow$  wrong (2 points deducted from 3 = 1 pt.)

$$3x - (40-x) = 85$$

$$3x - 40 + x = 85$$

$$4x - 40 = 85$$

$$+40 \quad +40$$

$$\frac{4x}{4} = \frac{125}{4}$$

$$x = 31.25$$

31.25 correct

8.75 wrong

$$40 - 31.25 = 8.75$$

9) (56, 80)  $\rightarrow$  (100, 100)

$$m = \frac{100-80}{100-56} = \frac{5}{11}$$

$$y = \frac{5}{11}x + b$$

$$y = \frac{5}{11}(100) + b \Rightarrow b = \frac{600}{11}$$

$$y = \frac{5}{11}x + \frac{600}{11}$$

If 60 is the lowest passing mark,

$$60 = \frac{5}{11}x + \frac{600}{11} \Rightarrow x = 12$$

So, a passing score on the original scale is 12.

10) 81 students

for every 8 students,  $\exists$  a chaperone

$$\begin{array}{r} 10.25 \\ 8 \overline{) 81} \end{array}$$

Groups of students:

8 8 8 8 8 8 8 8 = 72 students

$$\begin{array}{r} 81 \\ -72 \\ \hline 9 \end{array}$$

72 students, 9 chaperones

11)  $x+y = 625$  (# of sup bought)

$$12.50x + 9y = 7130$$

$$x+y = 625$$

$$\sqrt{x} = 625 - y$$

(Plug into 2nd equation)

(12)

Calcium:  $W = 3y$

$$W = 3y$$

$$X = 2y$$

Hydrogen:  $3x = 2z$

$$z = 3y$$

Phosphorus:  $x = 2y$

Oxygen:  $4x = 8y$

Hydrogen:  $3x = 2z$

$$3(2y) = 2z$$

$$6y = 2z$$

$$3y = z, \text{ so } W = z$$

Want  $W + X + y + z$  to be the smallest possible.

$$W + 2y + y + z$$

$$3y + 2y + y + 3y$$

$9y$  ← Must make minimum, so set  $y = 1$

$$\therefore W = 3(1) = 3$$

$$X = 2(1) = 2$$

$$y = 1$$

$$z = 3(1) = 3$$