FACTORING POLYNOMIALS

When working there are basically 3 cases to consider:

1) Factoring binomials (polynomials with 2 terms).
2) Factoring trinomials (polynomials with 3 terms).
3) Factoring by grouping (factoring polynomials with 4 terms).

Remember that in all cases, the first step in factoring a polynomial is to factor out the Greatest Common Factor (GCF).

Model Problems:

A) Factoring Binomials

There are 3 formulas to remember:

$a^2 - b^2 = (a + b)(a - b)$

$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$

$a^3 + b^3 = (a + b)(a^2 - ab + b^2)$

Notice that there isn't a formula to factor $a^2 + b^2$ using real numbers.

$$(1)x^2 - 4 = (x+2)(x-2)$$

$$(2)16x^4 - 1 = (4x^2 + 1)(4x^2 - 1) = (4x^2 + 1)(2x+1)(2x-1)$$

$$(3)8a^3 + 27 = ((2a)^3 + (3)^3) = (2a+3)[(2a)^2 - (2a)(3) + 3^2] = (2a+3)(4a^2 - 6a + 9)$$

$$(4)32x^6 - 4y^9 = 4(8x^6 - y^9) = 4[(2x^2)^3 - (y^3)^3] = 4(2x^2 - y^3)[(2x^2)^2 + (2x^2)(y^3) + (y^3)^2]$$

$$= 4(2x^2 - y^3)(4x^4 + 2x^2y^3 + y^6)$$

B) Factoring Trinomials ($ax^2 + bx + c$)

a) Case 1: $a = 1$

Example: Factor $x^2 - 3x - 28$.

The answer will be of the form $(x \quad)(x \quad)$, where the missing spaces are filled in with numbers that multiply to give -28 and add to give -3. In this case the answer is $(x-7)(x+4)$.

b) Case 2: $a \neq 1$

Example: Factor $6x^2 - 5x - 6$.

The most common method is the trial and error method. Here, the first terms must multiply to give $6x^2$, so the only ways to obtain this are either $(6x \quad)(x \quad)$ or $(3x \quad)(2x \quad)$. The second terms must multiply to give -6. To find the correct answer, you check whether the middle term $-5x$ works out correctly. It turns out that the correct answer is $(3x+2)(2x-3)$.

There are other methods you might have learned that are quicker and more efficient. Please use these methods if you find them easier.

C) Grouping

This method is used to factor polynomials with 4 terms. The idea is to factor out the GCF from the first two terms, and then factor out the GCF from the second pair of terms, and hopefully you will have the same expression in parenthesis. (It might happen that you have to rearrange the terms to factor correctly).

Factor $x^3+3x^2-16x-48$.

From the first two terms we can factor out x^2 and from the second two terms we can factor out -16: $x^3+3x^2-16x-48=x^2(x+3)-16(x+3)$. We now combine the terms in front of $(x+3)$ to obtain: $x^2(x+3)-16(x+3)=(x^2+16)(x+3)$. Now notice that the first factor is the difference of two squares. Altogether, then $x^3+3x^2-16x-48=(x+4)(x-4)(x+3)$.

Practice Exercises:

Factor completely:

1. y^2-9
2. x^3+8y^3
3. $2x^2-2x-12$
4. $x^3-3x^2+4x-12$
5. $3x^2+10x-8$
6. $5x^4-125$
7. $-x^2+6x-8$
8. $81a^9-3b^6$
9. $ac+ad+bc+bd$

Answers:

1. $(y+3)(y-3)$
2. $(x+2y)(x^2-2xy+4y^2)$
3. $2(x-3)(x+2)$
4. $(x^2+4)(x-3)$
5. $(3x-2)(x+4)$
6. $5(x^2+5)(x^2-5)$
7. $-(x-4)(x-2)$
8. $3(3a^3-b^2)(9a^6+3a^3b^2+b^4)$
9. $(a+b)(c+d)$