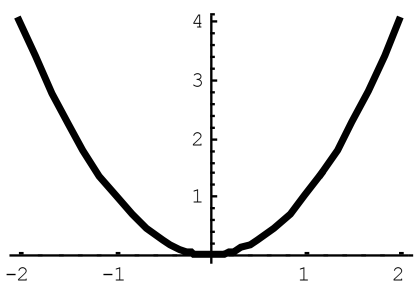
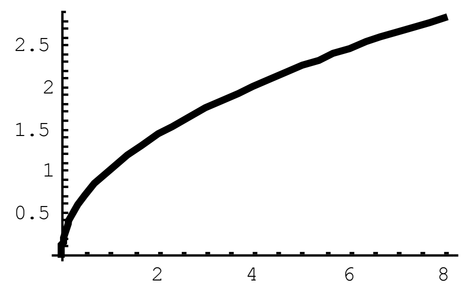
# Recognizing Some Functions and Their Graphs

There are several functions that you should be familiar with when entering a Calculus course. Below we list some of these graphs, and mention some of their properties.

$y = x^2$



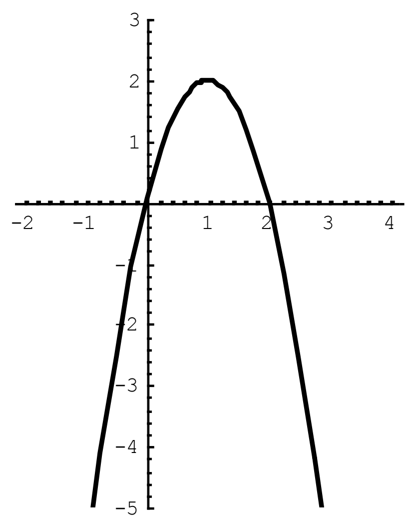

$y = \sqrt{x}$




$y = x^2$ is the basic quadratic function whose graph is a parabola, and $y = \sqrt{x}$ is the inverse for the part of the parabola that is greater than or equal to 0. (There isn't an inverse for the entire function since it is not one-to-one. The graph of $y = x^2$ doesn't pass the horizontal line test).

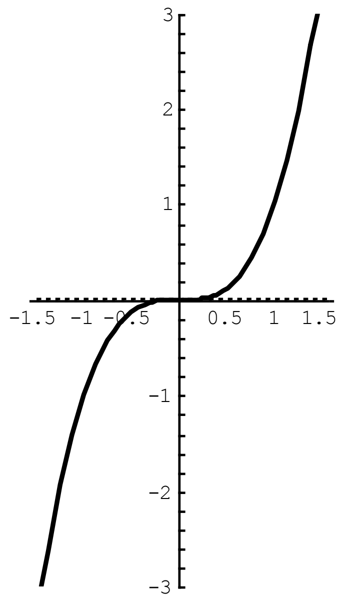
A more general quadratic function has the form $y = ax^2 + bx + c$. The vertex form of a quadratic function is $y = a(x-h)^2 + k$ where $(h,k)$ is the vertex. The parabola opens upwards if $a > 0$ and opens downwards if $a < 0$.

Example 1: Graph: $y = -2(x-1)^2 + 2$

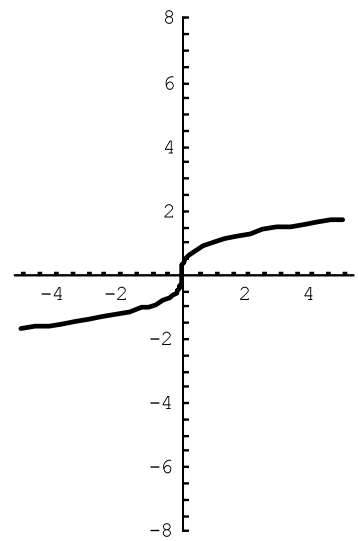


Example 2: Graph: (a) $y = x^3$ (b) $y = x^{\frac{1}{3}}$

(a)




(b)



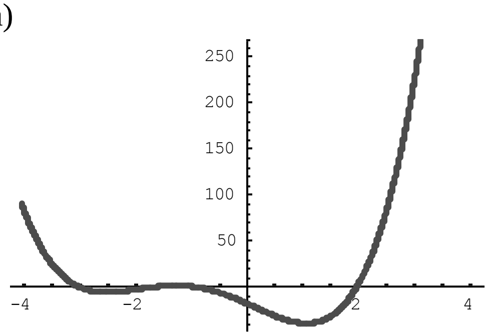

Notice that the graph of $y = x^3$ is flat when it crosses the origin. For its inverse, $y = x^{\frac{1}{3}}$, the curve seems to become vertical at the origin.

You should also be familiar with polynomial and rational functions. Rational functions are quotients of polynomials, and have additional features such as vertical, horizontal or slant (oblique) asymptotes.

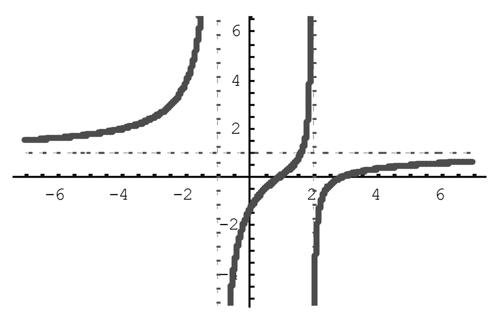
Example 3: Graph: (a) $p(x) = 2x^4 + 7x^3 - 4x^2 - 27x - 18$

(b) $r(x) = \dfrac{x^2 - 4x + 3}{x^2 - x - 2}$

(a)

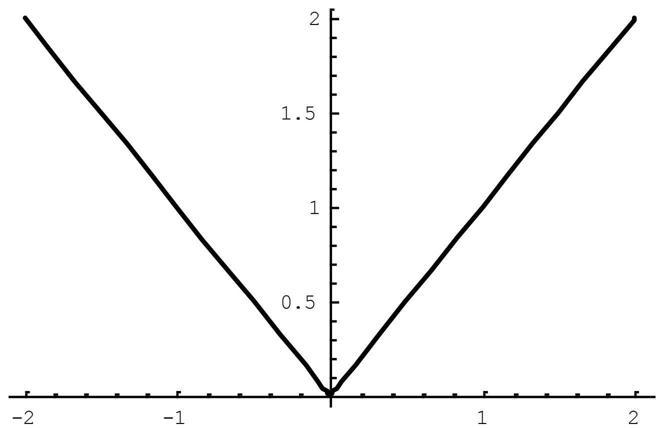



(b)



For the rational function, $x = -1$ and $x = 2$ are vertical asymptotes, and $y = 1$ is the horizontal asymptote.

$$y = |x|$$

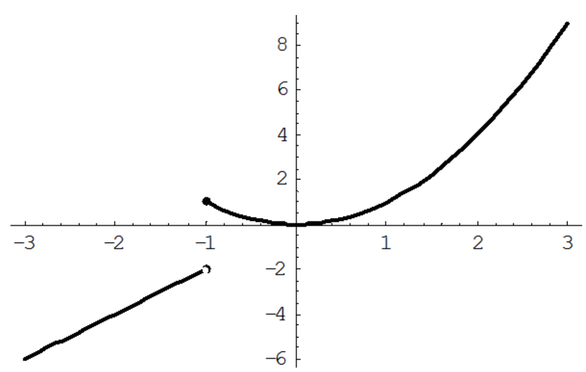



The distinguishing feature of the absolute value graph is the corner at the origin. The absolute value function is defined in a piecewise manner as

$$|x| = \begin{cases} -x, x < 0 \\ x^2, x \geq 0 \end{cases}$$

Another example of a piecewise-defined function is

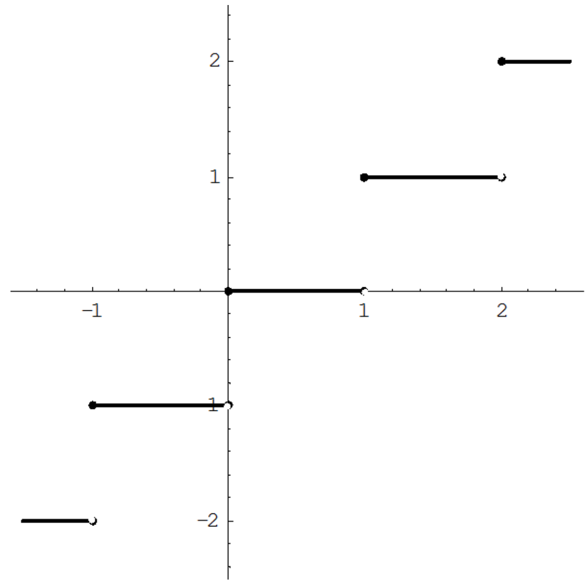
$$f(x) = \begin{cases} 2x, & x < -1 \\ x, & x \geq -1 \end{cases}$$




Piecewise-defined functions are often used to give examples of functions that have a jump somewhere in their graphs. For the example above, you can see there is a jump when $x = -1$, and there is a hole in the left part of the graph when $x = -1$.

A more complicated function, where there is a jump at each integer is given by the greatest integer function $[[x]]$.

$$[[x]] = \text{the largest integer less than or equal to the number } x$$




For example, $[\![3.45]\!] = 3$, $[\![-1.2]\!] = -2$, and $[\![10]\!] = 10$.

## Practice Exercises:

1. Graph the following functions.

   (a) $y = -2x + 4$

   (b) $y = x^2 + 4x + 3$ (hint: first put in vertex form by completing the square)

   (c) $y = 3(x - 1)^2 + 4$

   (d) $y = 2(x + 4)^2 - 3$

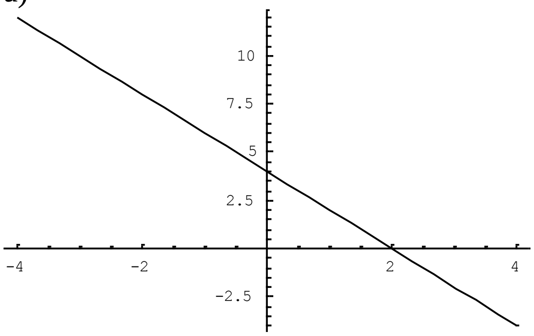
   (e) $y = -3x^2 - 9x + 6$

   (f) $f(x) = \begin{cases} x, x < 1 \\ x^2 - x, x \geq 1 \end{cases}$

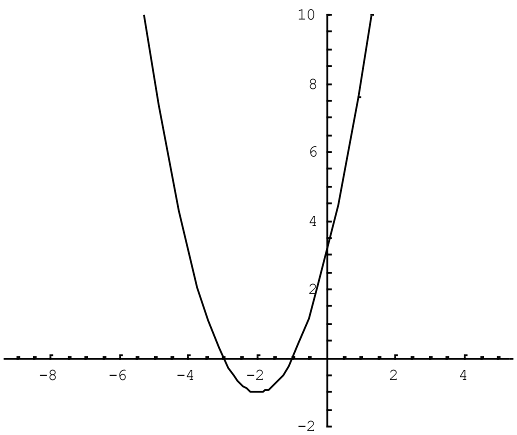
   (g) $y = 2x^3$

**SOLUTIONS:**

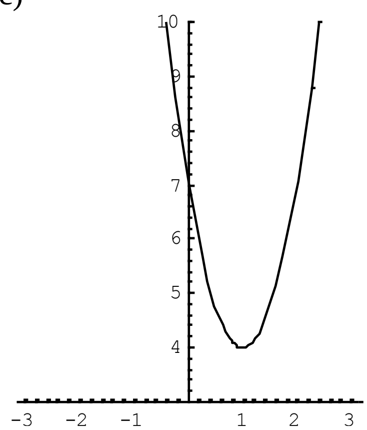
a)




b)




c)

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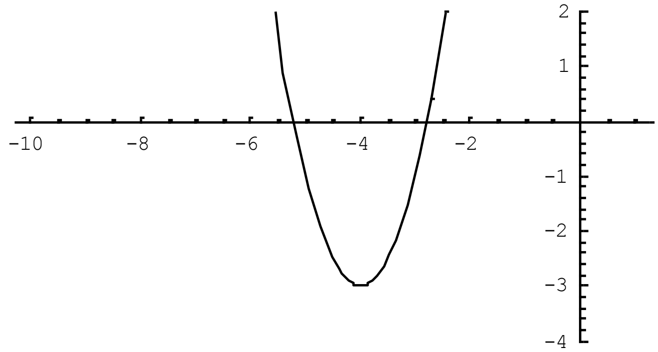
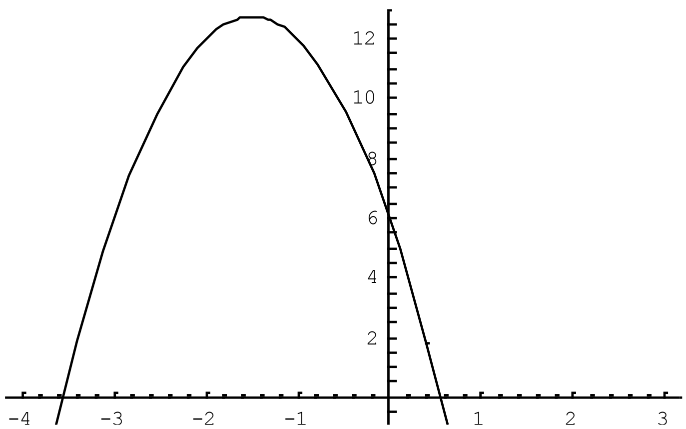
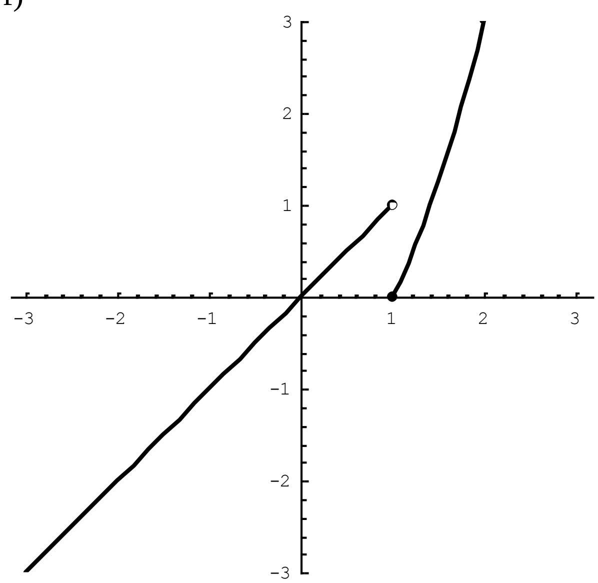


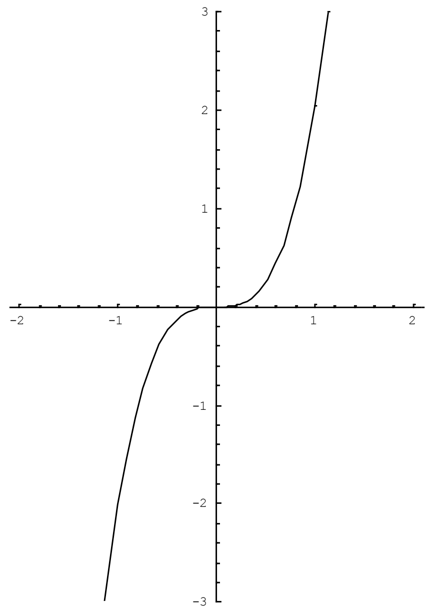

e)




f)




g)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2
1
-1
-2
-3
-2
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1
2