

SOLVING INEQUALITIES

EXAMPLE 1: Find the solution set for $k + 5 > 12$

SOLUTION:

$$\begin{array}{ll} k + 5 > 12 & \text{Original equation} \\ k + 5 + (-5) > 12 + (-5) & \text{Adding } -5 \text{ to both sides} \\ k > 7 & \text{Simplifying} \end{array}$$

Check:

$$\begin{array}{ll} k + 5 > 12 & k + 5 > 12 \\ 8 + 7 > 12 & 6 + 5 > 12 \\ 15 > 12 \text{ True} & 11 > 12 \text{ False} \end{array}$$

So the solution set for $k + 5 > 12$ consists of all real numbers greater than 7.

EXAMPLE 2: Find the solution set for $-3 + x + 5 < 7$

SOLUTION:

$$\begin{array}{ll} -3 + x + 5 < 7 & \text{Original equation} \\ 2 + x < 7 & \text{Simplifying the left-hand side} \\ 2 + x + (-2) < 7 + (-2) & \text{Adding } -2 \text{ to both sides} \\ x < 5 & \text{Simplifying} \end{array}$$

Check:

$$\begin{array}{ll} -3 + x + 5 < 7 & -3 + x + 5 < 7 \\ -3 + 3 + 5 < 7 & -3 + 6 + 5 < 7 \\ 5 < 7 \text{ True} & 8 < 7 \text{ False} \end{array}$$

So the solution set for $-3 + x + 5 < 7$ contains all real numbers less than 5.

EXAMPLE 3: Solve: $2(6x + 3) > 9x - 5$

SOLUTION

$$\begin{array}{ll} 12x + 6 + (-6) > 9x - 5 + (-6) & \text{Addition principle} \\ 12x > 9x - 11 & \text{Simplifying} \\ 12x + (-9x) > 9x - 11 + (-9x) & \text{Addition principle} \\ 3x > -11 & \text{Simplifying} \\ \left(\frac{1}{3}\right)(3x) > \left(\frac{1}{3}\right)(-11) & \text{Multiplying both sides by } \frac{1}{3} \\ x > -\frac{11}{3} = -3\frac{2}{3} & \text{Simplifying} \end{array}$$

EXAMPLE 4: Solve: $-4(y + 6) \leq 2y - 5 + 4(y - 7)$

SOLUTION

$-4(y + 6) \leq 2y - 5 + 4(y - 7)$	Original Equation
$-4y - 24 \leq 2y - 5 + 4y - 28$	Distributive property
$-4y - 24 \leq 6y - 33$	Combining like terms
$-4y - 24 + 24 \leq 6y - 33 + 24$	Adding 24 to both sides
$-4y \leq 6y - 9$	Simplifying
$-4y + (-6y) \leq 6y - 9 + (-6y)$	Adding $-6y$ to both sides
$-10y \leq -9$	Simplifying
$y \geq \frac{9}{10}$	Dividing by a negative value changes the inequality sign to its opposite

EXERCISES:

Solve using the addition principle.

- $-24 + x > 9$
- $x + 32 < 16$
- $-26 \leq y - 18$
- $-11t < -1144$
- $16x \geq -124$
- $2x - 3 - 9x > 10$
- $-5(x - 2) \geq 6x - 4 + 3(x + 5)$
- $-14r - 3(r + 9) > 7 + 2r$

SOLUTIONS:

- $x > 33$
- $x < -16$
- $y \geq -8$
- $t > 104$
- $x \geq -7\frac{3}{4}$
- $x < -1\frac{6}{7}$
- $x \leq \frac{-1}{14}$
- $r < \frac{-34}{19}$