Solving Systems of Linear Equations

There are two algebraic ways of solving a system of equations. Here is a reminder of each.

Example 1: Solve: $2x + 3y = 10$
$4x - 3y = 8$

Solution

If we add the left-hand sides and the right-hand sides of these equations, the *y* terms will drop out. We will be left with an equation in *x* only, which we can solve easily.

$$\begin{aligned} 2x + 3y &= 10 \\ 4x - 3y &= 8 \\ \hline 6x + 0 &= 18 \\ x &= 3 \end{aligned}$$

We now know that *x* = 3 is part of the solution of the system. We substitute 3 for x in either equation and solve for *y*.

$$\begin{aligned} 2x + 3y &= 10 \\ 2(3) + 3y &= 10 \\ 6 + 3y &= 10 \\ 3y &= 4 \\ y &= \tfrac{4}{3} \end{aligned}$$

So, the solution is $(3, \frac{4}{3})$.

Example 2: Solve: $2x + 9y = 49$
$5y = 31 - 3x$

Solution:

First we rewrite the equations with the variables in the same order on the same side. That makes everything easier.

$$\begin{aligned} 2x + 9y &= 49 \\ 3x + 5y &= 31 \end{aligned}$$

In order to be able to eliminate one variable, we want the coefficients of *x* or those of *y* to be additive inverses. The coefficients of *x* will be inverses if we multiply the first equation by 3 and the second equation by -2. Then we can add left-hand and right-hand sides, eliminating *x*, and solve for *y*. We get

$$\begin{aligned} 6x + 27y &= 147 && \text{Multiplying by 3} \\ -6x - 10y &= -62 && \text{Multiplying by -2} \\ \hline 17y &= 85 \\ y &= 5 \end{aligned}$$

Then we can substitute 5 for *y* in one of the original equations and solve it for *x*.

$$\begin{aligned} 3x + 5(5) &= 31 && \text{Substituting for x} \\ 3x + 25 &= 31 \\ 3x &= 6 \\ x &= 2 && \text{Simplifying} \end{aligned}$$

So, the solution is (2,5).

Example 3: Solve: $y = 2x + 4$
$6y + 3x = 54$

Solution

The first equation tells us that $y = 2x + 4$, so we can substitute $2x + 4$ for y in the second equation:

$$6y + 3x = 54$$
$$6(x + 4) + 3x = 54$$
$$12x + 24 + 3x = 54$$
$$15x + 24 = 54$$
$$15x = 30$$
$$x = 2$$

Then we can substitute 2 for *x* in one of the original equations and solve it for *y*. The first equation seems easiest.

$$y = 2x + 4$$
$$y = 2(2) + 4$$ Substituting for x
$$y = 8$$ Simplifying

We now have $(x, y) = (2, 8)$. To check, we substitute these values for x and y in the two equations.

$y = 2x + 4$	$6y + 3x = 54$
$8 = 2(2) + 4$	$6(8) + 3(2) = 54$
$8 = 4 + 4$	$48 + 6 = 54$
$8 = 8$	$54 = 54$

So, the solution is (2, 8).

Exercises: Use either method to solve the systems below.

1. $6x - 8y = 34$
   $y = 3x - 2$

2. $3x + 2y = 20$
   $x + y = 8$

3. $4x + 6y = 26$
   $6x - 2y = 28$

4. $y = x + 1$
   $y = -2x + 1$

5. $2x = 3y - 1$
   $y = 5$

Solutions

1. $(-1, -5)$ 2. $(4,4)$ 3. $(5,1)$ 4. $(0,1)$ 5. $(7,5)$