# Using Functional Notation

Sometimes, instead of writing an equation like $y = x^2 + 4$, textbooks use the notation $f(x) = x^2 + 4$. In other words, $y$ is replaced by $f(x)$. When this is done, this tells you that the equation is actually a function. That is, it is a special equation in which each $x$ (input) is assigned exactly one $y$ (output). It is not true that all equations determine such a relationship between the variables $x$ and $y$. For example, for the equation $x^2 + y^2 = 25$ (whose graph is a circle centered at the origin with radius 5), the number 3 corresponds to both 4 and -4. You can check that $(3,4)$ and $(3,-4)$ both satisfy the equation.

Functional notation is also useful in some calculations.

## Model Problems:

**Example 1**: Given $f(x) = x^2 + 4$, find:
(a) $f(2)$ (b) $f(-3)$ (c) $f(2+h)$

The function $f(x)$ says whatever is in the parentheses, square it and then add 4.
(a) $f(2) = 2^2 + 4 = 4 + 4 = 8$
(b) $f(-3) = (-3)^2 + 4 = 9 + 4 = 13$
(c) $f(2+h) = (2+h)^2 + 4 = (4 + 4h + h^2) + 4 = h^2 + 4h + 8$

In a first semester course in Calculus, you are asked to simplify such expressions as $\dfrac{f(2+h) - f(2)}{h}$ that are called Newton quotients or difference quotients.

**Example 2:** Given $f(x) = x^2 + 4$, simplify $\dfrac{f(-3+h) - f(-3)}{h}$.

$$\frac{f(-3+h) - f(-3)}{h} = \frac{(-3+h)^2 + 4 - ((-3)^2 + 4)}{h} = \frac{9 - 6h + h^2 + 4 - (9+4)}{h}$$
$$= \frac{13 - 6h + h^2 - 13}{h} = \frac{-6h + h^2}{h} = \frac{h(-6+h)}{h} = -6 + h.$$

For these calculations, when $f$ is a polynomial function, you should reach a point where the $h$ in the denominator can be cancelled out.

The following example is a little more difficult.

Example 3: Given $f(x) = \dfrac{1}{x}$, simplify $\dfrac{f(4+h) - f(4)}{h}$.

In the first step, we are combining the fractions in the numerator:

$$\frac{f(4+h)-f(4)}{h}=\frac{\frac{1}{4+h}-\frac{1}{4}}{h}=\frac{\frac{4-(4+h)}{4(4+h)}}{h}=\frac{\frac{-h}{4(4+h)}}{h}$$

In the second step, we are using the fact that $\frac{\frac{a}{b}}{c}=\frac{a}{bc}$. That is, the denominators get multiplied.

$$\frac{\frac{-h}{4(4+h)}}{h}=\frac{-h}{4h(4+h)}=-\frac{1}{4(4+h)}$$

## Practice Exercises:

1. Given $f(x)=2x-1$, find

(a) $f(4)$ (b) $f(4+h)$ (c) $\frac{f(4+h)-f(4)}{h}$

2. Given $f(x)=-x^2+2x$, find

(a) $f(-5)$ (b) $f(3+h)$ (c) $\frac{f(3+h)-f(3)}{h}$

## **Answers**:

1 (a) 7 (b) 7 + 2$h$ (c) 2

2a) -35 (b) $-h^2-4h-3$ (c) $-h$ - 4