

WORKING WITH PROPORTIONS

In the proportion $\frac{a}{b} = \frac{c}{d}$, a and d are called the extremes of the proportion, and b and c are called the means of the proportion. (Note: These “means” are not the same as the “average”)

If the proportion is written as $a : b = c : d$ we can see that a and d are at the extremes outside, whereas b and c are *between* the extremes.

The easiest way to determine whether a given proportion is true is by applying the **cross-products test**. In the proportion $a : b = c : d$, if $ad = bc$ then the proportion is true.

In fraction form, we can show the cross products as

$$\frac{a}{b} = \frac{c}{d}$$
$$b \times c$$
$$a \times d$$

MODEL PROBLEM 1

Determine whether $\frac{16}{8} = \frac{40}{20}$ is a true proportion.

SOLUTION

Using the cross-products test:

$$\frac{16}{8} = \frac{40}{20}$$
$$8 \times 4 = 320$$
$$16 \times 20 = 320$$

Because the product of the extremes is equal to the product of the means, the proportion is true.

Sometimes we are asked to find the value (of a mean or extreme) that will make a proportion true. The value that makes the cross products equal also makes the proportion true.

MODEL PROBLEM 2

Find the value of n that makes the following proportion true.

$$\frac{15}{16} = \frac{n}{9}$$

SOLUTION

We multiply to find the cross-products:

$$\text{Product of extremes: } 15 \times 9 = 135 \quad \text{Products of means: } 16 \times n$$

Since, in a true proportion, the product of the means is equal to the product of the extremes: $16 \times n = 135$

Solving for n :

$$\frac{16 \times n}{16} = \frac{135}{16}$$

$$n = \frac{135}{16}$$

$$n = 8\frac{7}{16}$$

To check, we replace n in the original proportion with $8\frac{7}{16}$ and cross-multiply.

$$\frac{15}{16} = \frac{8\frac{7}{16}}{9}$$

$$16 \times 8\frac{7}{16} = 135 \text{ and } 15 \times 9 = 135$$

PRACTICE:

Determine whether the ratios are proportional by using the cross-products test.

1. $\frac{0.5}{6} = \frac{4}{18}$

2. $\frac{16}{60} = \frac{4}{15}$

3. $\frac{\frac{2}{3}}{4} = \frac{3}{8}$

Find the missing value by using the cross-products test.

4. $\frac{7}{9} = \frac{1}{t}$

5. $\frac{3\frac{1}{2}}{4} = \frac{m}{7}$

6. $\frac{5}{6} = \frac{2.3}{r}$

7. $\frac{m}{6} = \frac{8}{1.5}$

ANSWERS:

1. No ($9 \neq 24$)

2. Yes

3. No ($12 \neq \frac{16}{3}$)

4. $t = \frac{9}{7}$

5. $m = 6.125$

6. $r = 2.76$

7. $t = 32$