

Working with Complex Fractions

A complex fraction is a fraction that contains a fraction (or fractions) in the numerator and/or denominator. To simplify a complex fraction is to write it as a simple fraction. An example of a

complex fraction is $\frac{1 + \frac{1}{x-2}}{1 - \frac{2}{x+1}}$.

There are two methods that are used to simplify complex fractions. The first method is usually quicker and is done by first multiplying each of the terms by the least common denominator of all the fractions that appear.

Model Problems:

Example 1:

$$\text{Simplify } \frac{\frac{3}{n+1} + \frac{1}{n}}{\frac{2}{n+1} + \frac{3}{n}}$$

The Least Common Denominator (LCD) is $n(n+1)$. We multiply each fraction in the numerator and denominator to eliminate the denominators.

$$\frac{n(n+1)\frac{3}{n+1} + \frac{1}{n}n(n+1)}{n(n+1)\frac{2}{n+1} + \frac{3}{n}n(n+1)} = \frac{3n+(n+1)}{2n+3(n+1)} = \frac{4n+1}{5n+3}$$

An alternate method which can be done is to first combine the fractions that appear in the numerator and denominator.

$$\frac{\frac{3}{n+1} + \frac{1}{n}}{\frac{2}{n+1} + \frac{3}{n}} = \frac{\frac{3n+(n+1)}{n(n+1)}}{\frac{2n+3(n+1)}{n(n+1)}} = \frac{4n+1}{5n+3} = \frac{4n+1}{n(n+1)} \cdot \frac{n(n+1)}{5n+3} = \frac{4n+1}{5n+3}$$

Practice:

$$1. \frac{x-4 + \frac{9}{2x+3}}{x+3 - \frac{5}{2x+3}}$$

$$2. \frac{1 + \frac{1}{y-2}}{1 - \frac{2}{y+1}}$$

$$3. \frac{\frac{x}{x+2} - \frac{x}{x-2}}{\frac{x}{x+2} + \frac{x}{x-2}}$$

Answers:

$$1. \frac{x-3}{x+4}$$

$$2. \frac{y+1}{y-2}$$

$$3. -\frac{2}{x}$$