

OPERATIONS ON POLYNOMIALS

Model Problem 1

Add: $(4x^2 + 3x - 6) + (-3x^2 - 12x - 8)$

Solution

First, we regroup the terms and then combine like terms.

$$(4x^2 + 3x - 6) + (-3x^2 - 12x - 8) = 4x^2 - 3x^2 + 3x - 12x - 6 - 8 = x^2 - 9x - 14$$

Model Problem 2

Simplify: $(3x^3 + 2x^2 - x + 4) - (x^2 - 5x)$

Solution

First, take the opposite of the second polynomial and change the operation to addition.

$$(3x^3 + 2x^2 - x + 4) + (-x^2 + 5x)$$

Then combine like terms.

$$3x^3 + 2x^2 - x^2 - x + 5x + 4 = 3x^3 + x^2 + 4x + 4$$

Model Problem 3

Multiply: $4x(2x - 4)$

Solution

First, distribute multiplication by $4x$ to each monomial inside the parentheses.

$$4x(2x - 4) = (4x)(2x) + (4x)(-4)$$

Then multiply monomials as indicated.

$$(4x)(2x) + (4x)(-4) = (4)(2)(x^{1+1}) + (4)(-4)x = 8x^2 - 16x$$

Model Problem 4

Multiply: $(2x + 5)(3x^2 - 3x + 4)$

Solution

Use the distributive property to multiply the second polynomial by each term of the first polynomial.

$$\begin{aligned}(2x + 5)(3x^2 - 3x + 4) &= 2x(3x^2 - 3x + 4) + 5(3x^2 - 3x + 4) \\ &= 2x(3x^2) + 2x(-3x) + (2x)(4) + 5(3x^2) + 5(-3x) + 5(4)\end{aligned}$$

Then simplifying, we get

$$6x^3 - 6x^2 + 8x + 15x^2 - 15x + 20 = 6x^3 + 9x^2 - 7x + 20$$

Model Problem 5

Multiply using FOIL: $(-7x + 3)(5x - 4)$

Solution

The FOIL method gives us

$$\begin{aligned}(-7x + 3)(5x - 4) &= (-7x)(5x) + (-7x)(-4) + 3(5x) + 3(-4) \\ &= -35x^2 + 28x + 15x - 12 \\ &= -35x^2 + 43x - 12\end{aligned}$$

Model Problem 6

Divide: $\frac{9x^2 - 15x + 3}{-3x}$

Solution

When the dividend has two or more terms, the entire numerator must be divided by the denominator.

$$\begin{aligned}\frac{9x^2 - 15x + 3}{-3x} &= \frac{9x^2}{-3x} + \frac{-15x}{-3x} + \frac{3x^0}{-3x} \\ &= -3x^{2-1} + 5x^{1-1} - x^{0-1} = -3x + 5x^0 - \frac{1}{x}\end{aligned}$$

PRACTICE

1) $(8x^2 + 5x + 5) + (7x^2 - 3x + 4)$

2) $(6y^2 + 2y - 4) + (-8y^2 - 6y + 3)$

3) $(n - 3) - (n + 4)$

4) $(7x^2 - 5) - (8x^2 + 3)$

5) $(7x - 4)(8x^2 + 3x - 9)$

6) $(-y + 7)(-6y^2 + 9y + 2)$

7) $(x^2 + 4)(x + 7)$

8) $(3x^2 + 5)(2x^2 - 4)$

9) $\frac{4x^4 + 12x^2 - 8}{-8x}$

10) $\frac{8r^3 - 8r + 6}{-4r^5}$

SOLUTIONS:

1) $15x^2 + 2x + 9$ 2) $-2y^2 - 4y - 1$ 3) -7 4) $-x^2 - 8$

5) $56x^3 - 11x^2 - 75x + 36$ 6) $6y^3 - 51y^2 + 61y + 14$

7) $x^3 + 7x^2 + 4x + 28$ 8) $6x^4 - 2x^2 - 20$ 9) $\frac{-x^3}{2} - \frac{3x}{2} + \frac{1}{x}$

10) $\frac{-2}{r^2} + \frac{2}{r^4} - \frac{3}{2r^5}$