# Solutions for Exam 2

## Problem #1:

Using the X-Y coordinate system drawn on the image of the four mass system, we can calculate two moments of inertia; one about a vertical axis which the system can rotate (rotation around the y-axis) and one about a horizontal axis which the system can rotate (rotation around the x-axis). Our formula is as follows:

$I = \sum_{i=1}^{N} m_i \, r_i^2$

($r_i$ is the distance of mass *i* from the specified rotational axis.)

**Part A**

$\mathbf{m_5 = 5;}$
$\mathbf{M_8 = 8;}$

$\mathbf{I_{about\ vertical} = m_5\ (-0.50)^2 + m_5\ (1.0)^2 + M_8\ (-0.50)^2 + M_8\ (1.0)^2}$

16.25

**Part B**

$\mathbf{I_{about\ horizontal} = m_5 \left(\frac{0.50}{2}\right)^2 + m_5 \left(\frac{0.50}{2}\right)^2 + M_8 \left(\frac{-0.50}{2}\right)^2 + M_8 \left(\frac{-0.50}{2}\right)^2}$

1.625

## Problem #2:

A sphere having a moment of inertia $I=(\frac{2}{5})MR^2$ rolls down an inclined plane. In order to solve for the angular velocity of the sphere when it reaches the bottom of the ramp we can use the conservation of mechanical energy principle. We have:

$\Delta$(Kinetic Energy) + $\Delta$(Potential Energy) = 0

Where the change in kinetic energy must be expressed in terms of the translational motion of the sphere AND the rotational motion of the sphere. The potential energy term is simply the potential due to the gravitational force acting on the sphere.

$$\left[\frac{1}{2} M_s\, v_f^2 - \frac{1}{2} M_s\, v_i^2\right] + \left[\frac{1}{2}\, I\omega_f^2 - \frac{1}{2}\, I\omega_i^2\right] + M_s g[H_f - H_i] = 0$$

Since the sphere is said to start from REST, our initial angular velocity magnitude and linear velocity magnitudes can be taken to be zero. We also take the final height to be 0. The above equation becomes:

$$\left[\frac{1}{2} M_s\, v_f^2\right] + \left[\frac{1}{2}\, I\omega_f^2\right] - M_s g[H_i] = 0$$

Rearrange and substitute $v_f = R_s * \omega_f$

$$M_s = 0.3;$$
$$g = 9.8;$$
$$v_f = R_s * \omega_f;$$
$$R_s = 0.75;$$
$$I_s = \frac{2}{5} * M_s * R_s^2;$$
$$H_i = 15;$$

$$\frac{1}{2} * M_s * R_s^2 * \omega_f^2 + \frac{1}{2} * I_s * \omega_f^2 = M_s * g * H_i;$$

$$\omega_f = \sqrt{\frac{(M_s * g * H_i)}{\left(\frac{1}{2} * M_s * R_s^2 + \frac{1}{2} * I_s\right)}};$$

$$\omega_f = \sqrt{\frac{10 * g * H_i}{7 * R_s^2}}$$

19.3218

The above expressions can be simplified in more than one way. For greater clarity, solve through this problem on your own.

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## Problem #3:

This problem is an example of the conservation of angular momentum. We have $L_i = L_f$, which is related to the conservation of linear momentum principle.

You could have left her initial rate of rotation in terms of revolutions per second or converted it to radians per second. The converstion to radians per second is as follows:

$$0.5 * 2 * \pi$$

3.14159

We do not divide by 60 since the initial rate is already expressed as revolutions per second, not minute.

$$I_1 = 7.0;$$
$$I_2 = 3.5;$$
$$\omega_1 = 3.14;$$

$$I_1 * \omega_1 = I_2 \, \omega_2;$$
$$\omega_2 = \frac{7.0 * \omega_1}{3.5}$$

6.28

The answer is 6.28 radians per second, which is 1 revolution per second.

## Problem #4:

**This problem is most easily solved from utilizing the concept of net torque and equilibrium conditions.**

**At equilibrium we have that** $\sum_{i=1}^{N} \tau_i = 0$

Take the rotational axis to be the point at which the ladder is touching the ground, 3 meters left over from the base of the wall.

First make a list of all the torques involved in the system:

$\tau_{\text{friction}} = 0$
$\tau_{\text{Cy}} = 0$
$\tau_{\text{painter}} = -\text{Mg} * \text{X}$
$\tau_{\text{wall}} = F_{\text{wall}} * (L_y)$
$\tau_{\text{ladder}} = -\text{mg} * \frac{L}{2}\text{Cos}[\theta]$

L=$\sqrt{L_x^2 + L_y^2}$ =5

$\tan^{-1}\left[\frac{L_y}{L_x}\right] = 53°$

We now have:

$\tau_{\text{wall}} + \tau_{\text{Cy}} + \tau_{\text{friction}} + \tau_{\text{painter}} + \tau_{\text{ladder}} = 0$

$F_{\text{wall}} * (L_y) + 0 + 0 - \text{Mg*X} - \text{mg} * \frac{\text{L}}{2} \text{Cos}[\theta] = 0$

$F_{\text{wall}} * (L_y) - \text{Mg*X} - \text{mg} * \frac{\text{L}}{2} \text{Cos}[\theta] = 0$

We need to solve for the force due to the wall in the x-direction using Newtons second law. F = ma. Since the coefficient of friction on the ground is 0.5, we can set up the ladders equilibrium condition in the x-direction between the force due to the wall and the force due to friction.

$F_{\text{friction}} - F_{\text{wall}} = m(0)$

$F_{\text{wall}} = 0.5(\text{m+M})\text{g}$

$m_{ladder} = 9;$
$M_{painter} = 70;$
$F_{wall} = 0.5 * (m_{ladder} + M_{painter}) * g;$
$\theta = 53°;$
$L = 5;$
$L_y = 4;$

$$0.5 * (m_{ladder} + M_{painter}) * g * L_y - m_{ladder} * g * \frac{L}{2} * \text{Cos}[\theta] = M_{painter} * g * \chi;$$

$$\chi = \frac{0.5 * (m_{ladder} + M_{painter}) * g * L_y - m_{ladder} * g * \frac{L}{2} * \text{Cos}[\theta]}{M_{painter} * g}$$

2.0637

This means that in order for the ladder not to slip, the painter can only walk a distance of 2.0637 meters in the x-direction away from the base of the ladder. His maximum height can be found using the tangent function.

$Y = \chi * \text{Tan}[\theta]$

2.73863

And the maximum total ‘Length’ he can travel along the ladder is...

$D = \sqrt{\chi^2 + Y^2}$

3.42913

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## Problem #5:

A 1500 kg boxcar traveling at 15.0 m/s strikes a second boxcar at rest. The two stick together and move off with a speed of 6.0 m/s. What is the mass of the second car? How much kinetic energy is lost in the collision?

This a conservation of linear momentum problem. It is a one-dimensional ‘sticking’ collision in which one boxcar is at rest while the other collides into it with an initial speed and the two end up traveling together with a new speed. We have the following expression:

$\Delta$(Momentum) = 0

$P_i = P_f$

$\text{Mass}_1 * u_1 + \text{Mass}_2 * u_2 = \text{Mass}_1 * u_{\text{final}} + \text{Mass}_2 * u_{\text{final}}$

Since Mass 2 is initially at rest we have:

$\text{Mass}_1 * u_1 = u_{\text{final}} * (\text{Mass}_1 + \text{Mass}_2)$

The mass of boxcar 2 can now be solved for since we have one equation and one unknown variable.

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Mass₁ = 1500;
u₁ = 15;
u_final = 6;
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$$Mass_2 = \frac{(Mass_1) * u_1}{u_{final}} - Mass_1$$

```
2250
```

The second part of the problem asks you to solve for the energy lost in the collision. We know that the system's energy is at a maximum initially before the collision occurs and at its lowest after the collision occurs (according to the information that we have).

$Energy_{initial}$ - $Energy_{final}$ = $Energy_{lost}$

$$Energy_{initial} = \frac{1}{2} * Mass_1 * u_1^2;$$

$$Energy_{final} = \frac{1}{2} * (Mass_1 + Mass_2) * u_{final}^2;$$

$$Energy_{initial} - Energy_{final}$$

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101 250
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We lose $1.0125*10^5$ joules of energy during the collision. This energy escapes the system most noticeably in the form of heat and sound.

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## Problem #6:

**Problem #6:** Two balls of equal mass masses $m_A = 0.06$ kg and $m_B = 0.06$ kg are suspended as show in the diagram below. The ball $m_A$ is pulled away to a 60° angle with the vertical and released.
(a) What is the velocity of the ball $m_A$ just before impact?
(b) What will be the maximum height ball $m_B$ after the elastic collision? Describe what happens to ball $m_A$ as a result of the collision.

In order to calculate the speed at which ball A hits into ball B with, we utilize the conservation of mechanical energy principle.

$$m_A * g * (L - L * \text{Cos}[\theta]) = \frac{1}{2} * m_A * v_c^2$$

Examine the geometry of the system to understand why the height of the ball A is L-L*Cos[$\theta$].

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m_A = 0.06;
L_s = 0.3;
θ₀ = 60 °;
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$$v_c = \sqrt{2 * g * (L_s - L_s * \text{Cos}[\theta_0])}$$

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1.71464
```

We calculate that Ball A hits into Ball B with a speed of 1.71 meters per second. Now what speed will Ball B begin its motion with? We utilize the concept of linear momentum to solve this second part of the problem:

Δ(Momentum) = 0

$m_A * v_c = m_A * v_{c_{\text{after}}} + m_B * v_{c_B}$

Since all the masses are the same, we can quickly solve this to find that:

$v_c = v_{c_{\text{after}}} + v_{c_B}$

After a bit of algebra we come to the conclusion that ball B recieves all of the kinetic energy from ball A after the collision, and so leaves with the same speed that ball A hit it with. This means that ball B will reach the height that ball A was dropped from, 0.15 meters.

Our final question to answer is, what happened to ball A? Ball A comes to a complete stop after hitting into ball B. It transferred all of its kinetic energy to ball B and therefore should have no kinetic energy left to move from the point of initial collision with ball B.

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## Problem #7:

**T**he period for the Earth about the Sun is T=365 days. We are asked to find the angular velocity of the earth about the Sun.

Knowing the relation $\omega = 2\pi$f and f = $\frac{1}{T}$

we can simply convert T into seconds and plug it into the relation.

$\mathbf{T_{seconds} = 365 * 24 * 60 * 60}$

31 536 000

$\omega_{\mathbf{Earth}} = \frac{\mathbf{2 * \pi}}{\mathbf{T_{seconds}}}$ **// N**

$1.99238 \times 10^{-7}$

## Problem #8:

A wheel 0.75 meter in diameter accelerates uniformly from 240 rpm to 420 rpm in 4.0 seconds. How far will a point on the edge of the wheel have traveled in meters in this time?

Convert both the initial and final angular velocities to radians per second and solve using either:

$\omega_f^2 - \omega_i^2 = 2\alpha\,(\Delta\theta)$

or

$\Delta\theta = \omega_i * (\Delta t) + \frac{1}{2} * \alpha * (\Delta t)^2$

$$r = \frac{0.75}{2};$$

$$\alpha_1 = \frac{420 * \frac{2*\pi}{60} - 240 * \frac{2*\pi}{60}}{(\Delta t)};$$

$$\Delta t = 4;$$

$$\Delta\theta = 240 * \frac{2*\pi}{60} * \Delta t + \frac{1}{2} * \alpha_1 * (\Delta t)^2 \text{ // N}$$

138.23

**Distance = r * Δθ**

51.8363

## Problem #9:

(a) What is the linear speed of a child seated 2.0 meter from the center?

Since the merry-go-round makes one complete revolution in 8 seconds, we can calculate our angular velocity to be...

$$\omega_{mgr} = \frac{2*\pi}{8};$$

Our relationship between linear speed and angular speed is as follows:

$v_{linear} = R_c * \omega_{mgr};$
$R_c = 2;$

$v_{linear}$ // N

1.5708

(b) What is her radial acceleration?

We can use the relationship for radial acceleration:

$$a_R = \frac{(v_{linear})^2}{R_c} \text{ // N}$$

1.2337

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## Problem #10:

How much work is required to stop a proton that has a mass m=1.67 × $10^{-27}$ kg and is moving with a speed 3.0 × $10^6$ of m/s?

The amount of work required to stop the proton from moving would be equivalent to its overall change in kinetic energy.

$$\text{Work} = 0 - \frac{1}{2} * \left(1.67 * 10^{-27}\right) * \left(3.0 * 10^{6}\right)^2$$

$-7.515 \times 10^{-15}$

Thus it would take $-7.515 \times 10^{-15}$ joules to stop the proton.

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## Problem #11:

Newton's second law of motion is not valid in a rotating coordinate system. The centrifugal force is a fictional force which is included in the usual Newton's second law of motion so it works in rotating coordinate systems. The centrifugal force is in the outward radial direction while the centripetal force is in the inward radial direction. The centripetal force is a real force like for example gravity.

Everything in this paragraph is TRUE.

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## Problem #12:

Con Edison sells electricity by charging a $0.50 per KiloWatt-Hour. The KiloWatt-Hour is a unit of power or energy?

This problem can be solved using dimensional analysis.

A kiloWatt is a unit of POWER, and POWER is the rate of work with respect to time. The unit of work is joules, which is a unite of energy, and so we have...

(kiloWatt)*(Hour)

$\left(\frac{\text{Energy}}{\text{Time}}\right) * (\text{Time})$

ENERGY

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## Problem #13:

Radians and Degrees are two different units used to measure angular distance $\theta$. The Degree measurement of the size of an angle is independent of the size of the radius of the circular motion AND so too is the radian measure of an angle independent of the size of the radius of the circular motion."

**TRUE**

## Problem #14:

**Problem #14**: Suppose a mass slides down a hill as pictured below. Suppose the M=7 kg mass is released from rest at the top of the hill which is a height H=3.5 meters above the reference level. How fast is the mass going when it reach the bottom of the hill which is h=1.7 meters above the reference level? Assume there is no friction force. Do not use Newton's laws of motion to solve this problem.

You can use the conservation of mechanical energy principle to solve this problem:

Δ(Kinetic Energy) + Δ(Potential Energy) = 0

$\frac{1}{2} m(v_f{}^2 - v_i{}^2) + \text{mg(H-h)} = 0$

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H = 3.5;
h = 1.7;
m = 7;
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$V_{Final} = \sqrt{2 * g * (H - h)}$

```
5.9397
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## Problem #15:

Suppose the tangential velocity of the point P increases. Circle all of the following that are TRUE.

(a) The angular velocity increases.

$V_{\text{tangential}} = R\,(\omega)$

(b) The angular acceleration is not zero.

If the tangential velocity is increasing, then there is a tangential acceleration, which means the angular accelaration is NOT zero:

$a_{\text{tangential}} = R\,(\alpha)$

(c) A torque must have been applied to the circle.

$\tau = \text{I}\,\alpha$

(d) A tangential force must have been applied to the circle.

$\text{F} = \text{m}\,a_{\text{tangential}}$

(e) The radial acceleration increases.

Since the tangential velocity is increasing, the radial acceleration increases as well due to the relation:

$a_{\text{radial}} = \frac{(v_{\text{tangential}})^2}{R}$

ALL OF THE OPTIONS ARE TRUE