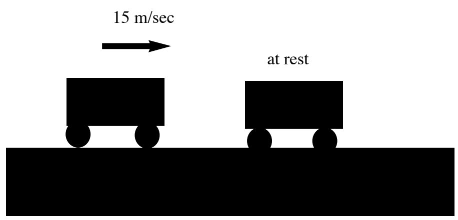
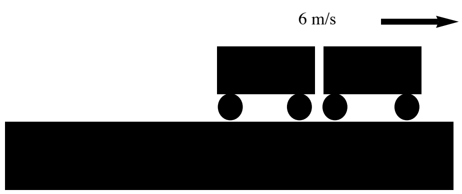
Chapter #7 Giancoli 6th edition Problem Solutions

- **Problem #8**

QUESTION: A 9300 kg boxcar traveling at 15.0 m/s strikes a second boxcar at rest. The two stick together and move off with a speed of 6.0 m/s. What is the mass of the second car?
ANSWER:



Before Collision



After Collision

Momentum is conserved since there is no external force acting on the system of two boxcars in the horizontal direction. (There is an external force (gravity) in the y-direction but there is no motion in the y direction.) Momentum is NOT conserved for each boxcar separately. The two boxcars stick together and this usually means energy is NOT conserved or at least cannot be assumed to be conserved.

The initial momentum is $p_0 = 9300\,\text{kg} \times 15\,m/s$ and the final momentum is $p_f = (9300\,\text{kg} + M) \times 6\,m/s$ if we knew the mass M of the second boxcar. Conservation of momentum means $p_0 = p_f$ that is

$$9300\text{ kg} \times 15\text{ m/s} = (9300\text{ kg} + \text{M}) \times 6\text{ m/s}$$

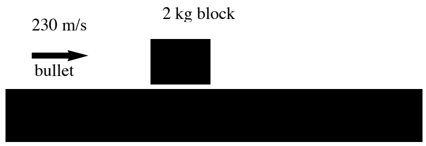
$$139500\text{ kg-m/s} = 55800\text{ kg-m/s} + \text{M} \times 6\text{ m/s}$$

$$\text{M}=13{,}950\text{ kg.}$$

```
9300 * 15

139 500

930 086

9300 * 6

55 800

(139 500 - 55 800) / 6.

13 950.
```

- **Problem #12:**

QUESTION: A 23-gm (=0.023 kg) bullet traveling 230 m/s penetrates at 2.0 kg block of wood and emerges cleanly at 170 m/s If the block is stationary on a frictionless surface when hit, how fast does it move after the bullet emerges?
ANSWER:



Before

After

Momentum is conserved for the system of the bullet and 2.0 kg block since there is no external force acting on the system in the horizontal direction. (Gravity is an external force which acts in the vertical direction but there is no motion in the vertical direction.) Momentum is not conserved for the bullet separately or the block of wood separately.

The initial momentum of the system (bullet plus block) is $p_0 = 0.023\,\text{kg} \times 230\,m/s = 5.29$ kg-m/s

```
0.023 * 230
```

```
5.29
```

The final momentum of the system $p_f = 0.023\,\text{kg} \times 170\,m/s + 2.0\,\text{kg} \times V_f = 3.91$ kg-m/s + 2 kg × V_f

```
170 * 0.023
```

```
3.91
```

Assuming conservation of momentum $p_0 = p_f$ means that 5.29 kg-m/s= 3.91 kg-m/s + 2 kg × V_f
Solving for V_f yields

$$V_f = \frac{5.29\,\text{kg}-m/s - 3.91\,\text{kg}-m/s}{2\,\text{kg}} = 0.69\ \text{m/s}$$

```
5.29 - 3.91
-----------
    2.0
```

```
0.69
```

- **Problem #16**

QUESTION: A 12 kg hammer strikes a nail at a velocity of 8.5 m/s and comes to rest in a time interval of 8.0 sec.
(a) What is the impulse given to the nail?
(b) What is the average force acting on the nail?

ANSWER: The change in momentum of the hammer is $\Delta p = p_f - p_0$ where the final momentum of the hammer is zero p_f=0 since the hammer comes to rest. The initial momentum of the hammer is p_0=12 kg × 8.5 m/s = 102 kg-m/s. So the change in momentum of the hammer is $\Delta p = p_f - p_0$ = (0 - 102 kg-m/s) = -102 kg-m/s. (The positive x direction is in the direction of the motion of the hammer so the initial velocity of the hammer 8.5 m/s is positive.) The change in momentum of the hammer equals the impulse due to the nail on the hammer which by Newton's 2nd law

$$\text{impulse of the nail on the hammer} = F\,\Delta t = \Delta p = -102\ \text{Nt-sec}$$

where F is the average force of the nail on the hammer and Δt=8.0 msec. (msec=10^{-3} sec.) It is the force of the nail on the hammer that changes the momentum of the hammer. (The impulse of the hammer on the nail is equal in size to this but opposite in direction due to Newton's 3rd law and because the time of contact Δt is the same for the hammer and the nail.) So the average force of the nail on the hammer F is

$$F = \frac{\Delta p}{\Delta t} = \frac{-102\ \text{kg}-m/s}{8.0 \times 10^{-3}\ s} = -\ 12{,}750\ \text{Nt.}$$

By Newton's 3rd law, the force of the hammer on the nail is equal in size and opposite in direction to F so the force of the hammer on the nail is +12,750 Nt. and this is in the positive x direction as expected.

```
12 * 8.5
```

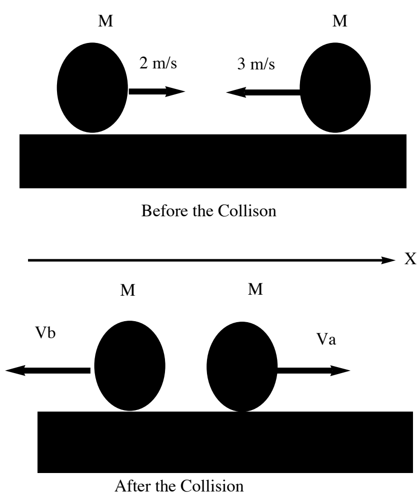
```
102.
```

```
102. / .008.
```

```
12750
```

- **Problem #24**

QUESTION: Two billiard balls of equal mass undergo a perfectly elastic head-on collision. If one ball's initial speed was 2.0 m/s and the other's was 3.0 m/s in the opposite direction, what will be their speeds after the collision?
ANSWER:



Va is the velocity of the ball on the right after the collision and Vb is the velocity of the ball on the left after the collision. Assuming momentum is conserved

$$M \times 2\text{m/s} + M \times (-3\ \text{m/s}) = M\times(-Vb) + M\times Va$$

and after cancellation of M's we get

$$2 - 3 = -Vb + Va \quad \text{or} \quad -1 = -Vb + Va = Vb-1 \quad \text{(Equation \#1)}$$

The positive x direction is to the right and it is assumed the velocity of the ball on the right is positive after the collision so Va is positive and the ball on the left is assumed moving to the left with negative velocity (-Vb) after the collision since Vb>0. If these assumptions are incorrect, in the process of solving the problem Va and/or Vb may turn out to be negative and that will tell us our assumption(s) is/are incorrect and the balls are actually moving in the reverse directions after the collision.

Assuming energy is conserved we get

$$\frac{1}{2} M \times (2\ m/s)^2 + \frac{1}{2} M \times (-3\ m/s)^2 = \frac{1}{2} M\times(-Vb)^2 + \frac{1}{2} M\times Va^2$$

and after cancellation of Ms and 1/2 we get

$$4 + 9 = Vb^2 + Va^2 \quad \text{or} \quad 13 = Vb^2 + Va^2 \quad \text{(Equation \#2)}$$

Equations #1 and #2 have two unknowns which we solve for by first writing equation #1 as Va = Vb -1 and using this to eliminate Va in equation #2 obtaining

$$13 = (Vb - 1)^2 + Vb^2 \quad \text{or} \quad 2\,Vb^2 - 2\,Vb - 12 = 0$$

This is a quadratic equation for Vb which is easily solved to get

$$Vb = \frac{2 \pm \sqrt{4 + 4*2*12}}{2*2}$$

```
(2 + Sqrt[4 + 4*2*12]) / (2*2)
```

3

```
(2 - Sqrt[4 + 4*2*12]) / (2*2)
```

-2

Using *Mathematica* to check the results above:

```
Solve[{2 * X * X - 2 * X - 12 == 0}, {X}]
```

{{X → -2}, {X → 3}}

So there are two possible solution for Vb:

1st solution is Vb = -2 m/s and returning to Va = Vb-1 we get Va =-3m/s.
Since both Va and Vb are minus it means we guessed wrong about the directions after the collisions. More importantly, this result says the particle b on the left passes through the particle a on the right and particle b has it's velocity unchanged. (and similarly for particle a). This is not possible physically *so we reject this solution*.

2nd solution is Vb= 3m/s and returning to Va = Vb-1 we get Va =2m/s.
Since both Va and Vb are positive we know we guessed right as to their directions. The 1st solution means that after the collision particle a has the same velocity it started with and the same for particle b. The only way this could happen is if the particles pass through each other but this is not physically possible. So the 1st solution is ruled out on physical grounds.

As a check *Mathematica* can be used to solve the two equations:

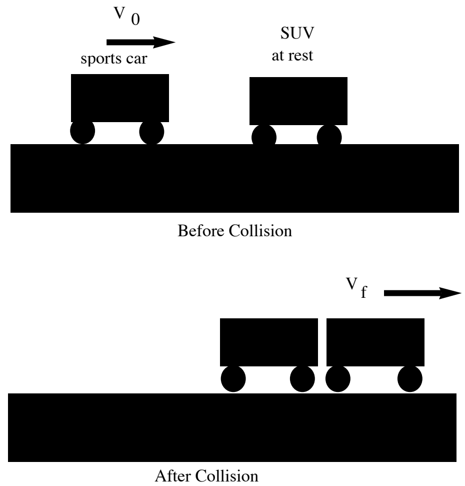
```
Solve[{2 - 3 == -Vb + Va, 13 == Vb^2 + Va^2}, {Va, Vb}]
```

{{Va → -3, Vb → -2}, {Va → 2, Vb → 3}}

- **Problem #35**

QUESTION: A m=920 kg sports car collides into the rear end of a M=2300 kg SUV stopped at a red light. The bumpers lock, the brakes are locked, and the two cars skid forward 2.8 m before stopping. The police officer, knowing that the coefficient of kinetic friction between tires and road is 0.80 calculates the speed of the sports car at impact. What was that speed?

ANSWER: The physical problem is similar to problem #12 above and we use the same diagram except the masses are different and the initial speed V_0 of the sports car is not known in problem #35.



Momentum is conserved since there is no external force acting on the system (the sports car plus SUV) in the horizontal direction. (There is an external force (gravity) in the y-direction but there is no motion in the y direction.) Momentum is NOT conserved for the sports car and SUV separately. The sports car and SUV stick together and this usually means energy is NOT conserved or at least cannot be assumed conserved. The kinetic energy lost winds up as heat energy so energy is still conserved just kinetic energy of the center of mass is NOT.

The initial momentum is $p_0 = m\ V_0 = 920\,\text{kg} \times V_0$ and the final momentum is $p_f = (m+M) \times V_f = (920\ \text{kg} + 2300\ \text{kg}) \times V_f$. Conservation of momentum means $p_0 = p_f$ so in this case, this means

$$920\,\text{kg} \times V_0 = (920\,\text{kg} + 2300\,\text{kg}) \times V_f$$

$$V_0 = 3.5\ V_f$$

$$\frac{920 + 2300}{920.}$$

3.5

So if we knew the final velocity V_f of the sports car plus SUV right after the collision, then we would find the initial speed of the sports car V_0. The sports car plus SUV system travels a distance X=2.8 m before stopping. Assume the initial kinetic energy of the sports car plus SUV equals the work done against friction μNx where N=(m+M)g is the normal force of the sports car plus SUV system and μ=0.80 is the coefficient of friction. Thus

$$\frac{1}{2}(m + M) \times V_f^{\,2} = \mu\,(m+M)\text{g x}$$

The mass (m+M) cancels in the above equation and we solve for V_f obtaining

$$V_f = \sqrt{2\,\mu\,g\,x} = \sqrt{2 \times 0.80 \times 9.8\,m/s^2 \times 2.8\,m} = 6.63\ \text{m/s}$$

$\sqrt{2 * 0.80 * 9.8 * 2.8}$

6.62601

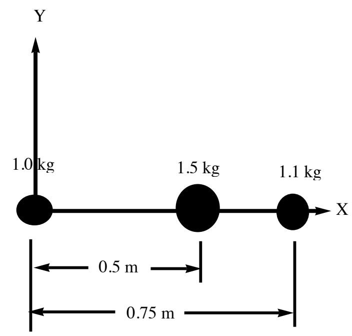
This value of V_f=6.63 m/s can now be used to compute V_0 using the result from conservation of momentum V_0 = 3.5 V_f so that

$V_0 = 3.5 * 6.63\, m/s$

$\frac{23.205\ \text{m}}{\text{s}}$

- **Problem #46**

Calculate the center of mass or COM of the three-mass system shown below. Specify the COM relative the left-hand 1.0 kg mass



$$X = \frac{1.0\,\text{kg} \times 0\, m + 1.5\,\text{kg} \times 0.5\, m + 1.1\,\text{kg} \times 0.75\, m}{1.0\,\text{kg} + 1.5\,\text{kg} + 1.1\,\text{kg}} = 0.44\ \text{m}$$

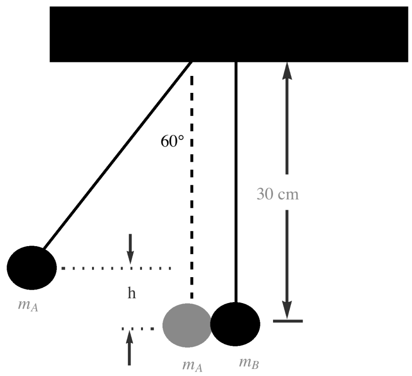
since using *Mathematica* to compute the numbers yields

$$\frac{1 * 0 + 1.5 * 0.5 + 1.1 * 0.75}{1.0 + 1.5 + 1.1}$$

0.4375

- **Problem # 76**

Two balls of masses $m_A = 40$ gm $=$ 0.04 kg and $m_B = 60$ gm $=$ 0.06 kg are suspended as show in the diagram below. The lighter ball is pulled away to a 60° angle with the vertical and released.
(a) What is the velocity of the lighter ball before impact?
(b) What will be the maximum height of each ball after the elastic collision?



Energy is conserved from the release of mass m_A until just before it hits mass m_B. So the initial potential energy equals the kinetic energy of m_A just before impact when it has a velocity V

$$m_A\, g\, h = \frac{1}{2} m_A V^2 \quad \text{or} \quad V = \sqrt{2\, g\, h}$$

The height h the mass rises vertically is given by

$$h = 30 \text{ cm} - 30 \text{ cm} \times \text{Cos}[60°] = 15 \text{ cm} = 0.15 \text{ m}$$

and using this to compute V yields

$$V = \sqrt{2\, g\, h} = 1.71 \text{ m/s}$$

Using *Mathematica* to check the calculation

```
h = 0.30 - 0.30 * Cos[60 °]
```

```
0.15
```

```
g = 9.8;
V = √(2 * g * h)
```

```
1.71464
```

Part b: Momentum is conserved when mass m_A hits mass m_B so we write $p_0 = m\, V$ as the initial momentum and $p_f = m_A V_A + m_B V_B$ where V_A is the velocity of m_A after the collision and V_B is the velocity of m_B after the collision. Conservation of momentum yields

$$m_A V = m_A V_A + m_B V_B$$

Using V=17.1 m/s in equation above together with $m_A = 40$ gm $= 0.04$ kg and $m_B = 60$ gm $= 0.06$ kg yields

$$0.04 \text{ kg} \times 1.71 \text{ m/s} = 0.04 \text{ kg} \times V_A + 0.06 \text{ kg} \times V_B$$

Simplifying this equation

$$0.0684 = 0.04 \times V_A + 0.06 \times V_B \quad \text{equation \#1}$$

Assume kinetic energy K.E. is conserved for the collision process of m_A with m_B so

$$\frac{1}{2}\ m_A V^2 = \frac{1}{2}\ m_A V_A{}^2 + \frac{1}{2}\ m_B V_B{}^2$$

Utilization of the numerical values for m_A, m_B, and V in the above equation yields

$$0.04\text{ kg} \times (1.71\, m/s)^2 = 0.04\text{ kg} \times V_A{}^2 + 0.06\text{ kg} \times V_B{}^2$$

or a little more simply

$$0.118 = 0.04 \times V_A{}^2 + 0.06 \times V_B{}^2 \quad \text{equation \#2}$$

Equation #1 and #2 have two unknowns V_A and V_B. First solve equation #1 for $V_A = \frac{0.684 - 0.06\, V_B}{0.04}$ then use this last equation to eliminate V_A in equation #2 and what results is a quadratic equation for V_A which is easily solved. *Mathematica* can do the work for us:

```
Solve[{0.068 == 0.04 * VA + 0.06 * VB, 0.118 == 0.04 * VA^2 + 0.06 * VB^2}, {VA, VB}]
{{VA → -0.357497, VB → 1.37166}, {VA → 1.7175, VB → -0.0116647}}
```

The first solution VA=-0.36 m/s and VB=1.37 m/s is the valid solution. The second solution makes no physical sense since it says mass m_A passes through mass m_B and m_A continues on with the same velocity it had before the collision that is, 1.71 m/s and mass m_B remains at rest (0.01 m/s is zero in comparison with 1.71 m/s) throughout. Mass A will rise up a distance hA = .0066 m determined from conservation of energy $\frac{1}{2}\, m_A V_A{}^2 = m_A\, g\, h_A$ which after cancellation yields $h_A = \frac{V_A{}^2}{2g}$ =0.0066 m and something similar for mass B, that is, hB=0.096 m. *Mathematica* gives for the two heights

```
g = 9.8;
hA = 0.36^2 / (2 * g)
0.00661224
hB = 1.37^2 / (2 * g)
0.0957602
```