Problem Solutions Chapter #8 Giancoli 6th Edition

- **Problem #8:**

QUESTION: A rotating merry - go - round makes one complete revolution in 4.0 seconds (a) What is the linear speed of a child seated 1.2 meter from the center? (b) What is her acceleration?
ANSWER: The linear or tangential velocity V=ωR where R=1.2 m is the distance to the axis of rotation. The period of revolution is T=4.0 sec/cycle so the frequency f=1/4 cycles/sec= 0.25 Hertz an the angular frequency in radians per second is ω=2πf=1.57 Rad/s. The linear speed V=ωR=1.57 Rad/s × 1.2 m =1.9 m/s. The tangential acceleration is zero because it seems ω is constant (and α the angular acceleration is zero). The radial acceleration is V_R=V^2/R=ω^2R=2.96 m/s^2 in the radial direction toward the center of the merry-go-round.

```
T = 4.0;
f = 1 / T;
ω = 2 * π * f
```

1.5708

```
R = 1.2;
V = ω * R
```

1.88496

```
a = V^2 / R
```

2.96088

- **Problem #9**

Calculate the angular velocity of the Earth (a) as it orbits the Sun and (b) about its axis.
ANSWER: T=365 days/cycle for the Earth going about the Sun. f=1/T is the frequency in cycles/sec and ω=2πf is the angular frequency is Rad/sec and ω=1.2 × 10^{-7} Rad/s.

```
T = 365. * 24 * 60 * 60.;
f = 1 / T;
ω = 2 * π * f
```

1.99238×10^{-7}

PART B: T=24 hours per day and this corresponds to ω=7.27 × 10^{-5} Rad/s.

```
T = 24. * 60 * 60;
f = 1 / T;
ω = 2 * π * f
```

0.0000727221

- **Problem #16**

An automobile engine slows down from 4500 rpm to 1200 rpm in 2.5 seconds. Calculate (a) its angular acceleration α assuming it is constant. 2π Rad = 1 Rev

```
ωf = 1200. * (2 * π);
ω0 = 4500. * (2 * π);
t = 2.5;
α = (ωf - ω0) / t
```

-8293.8

The minus sign means there is a deceleration or slowing down.
Part (b) Calculate the total number of revolutions is makes in the time t=2.5 sec.
ANSWER:

```
θ0 = 0;
ω0 = 4500. * (2 * π);
α = -8290.;
t = 2.5;
θf = θ0 + ω0 * t + 1/2 * α * t^2
```

```
44 779.6
```

If there had been no deceleration then θ=ωt=70686 Rad which is much larger

```
ω0 * t
```

```
70 685.8
```

- **Problem #18**

A wheel 33 cm in diameter accelerates uniformly from 240 rpm to 360 rpm in 6.5 seconds. How far will a point on the edge of the wheel have traveled in this time?
ANSWER: The angular acceleration α=1.9 Rad/sec^2 The wheel rotates through angle θ=204 Rad in that time. The total distance around the circumference is S= 33.7 meters

```
ω0 = 240. * (2 * π)/60.;
ωf = 360. * (2 * π)/60.;
t = 6.5;
α = (ωf - ω0)/t
```

```
1.93329
```

```
θ = ω0 * t + (1 / 2) * α * t^2
```

```
204.204
```

The distance above is in Radians. In terms of Rev measure see below:

```
204 / (2. * π)
```

```
32.4676
```

```
R = (0.33 / 2);
S = R * θ
```

```
33.6936
```

- **Problem #25**

Two blocks each having mass m are attached to the ends of a mass-less bar as in the figure below:

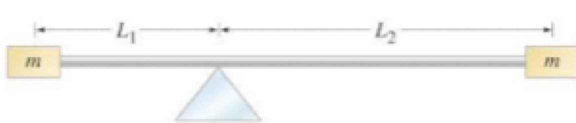

FIGURE 8–40 Problem 25.

Initially the bar is held at rest but then it is released so it can rotate. What is the torque acting on the rod?
ANSWER: τ=mg L_1- m g L_2 where clockwise is negative for torques τ.

- **Problem #29**

A small 650 gm ball on the end of a thin, light rod is rotated in a horizontal circle of radius 1.2 m Calculate

Part A: The moment of inertia of the ball about the center of the circle.
ANSWER: $\mathbb{I}$=mr^2=0.94 kg-m^2

```
m = 650 / 1000.;
r = 1.2;
𝕀 = m * r^2
```

```
0.936
```

Part B; Calculate the torque τ needed to keep the ball moving in a circle with the angular velocity ω constant if the force of air resistance if F=0.02 Nt.
ANSWER: The torque produced by the air resistance is τ=F r

```
F = 0.02;
r = 1.2;
τ = F * r
```

```
0.024
```

The net or total torque has to be zero for ω to be constant since α=0 and $\sum\tau=\mathbb{I}\alpha$ and thus $\sum\tau=0$. A torque opposite to the air resistance torque and must be applied and have size 0.024 Nt-m.

- **Problem #31**

Calculate the moment of inertia of the array of point masses shown in figure 8-43 below. Assume m=1.8 kg and M=3.1 kg and the objects are wired together by very light, rigid pieces of wire. The array is rectangular with the dimensions indicated. The horizontal axis splits the 0.50 m distance in half. The vertical axis is 0.50 m from the left side of the rectangle.

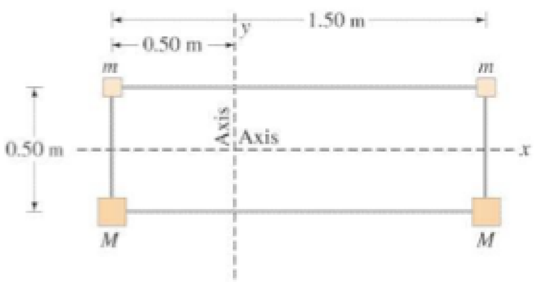

FIGURE 8–43 Problem 31.

Part A: What is the moment of inertia about the vertical axis?

$\mathbb{I}$ = 1.8 kg × (0.50 m)^2 + 1.8 kg × (1.50 m)^2 + 3.1 kg × (0.50 m)^2 + 3.1 kg × (1.50 m)^2 = 12.25 kg-m^2

```
𝕀 = 1.8 * (0.50 )^2 + 1.8 * (1.50 )^2 + 3.1 * (0.50)^2 + 3.1 * (1.50)^2
```

```
12.25
```

Part B: What is the moment of inertia about the horizontal axis?

$\mathbb{I}$ = 1.8 kg × (0.25 m)^2 + 1.8 kg × (1.50 m)^2 + 3.1 kg × (0.25 m)^2 + 3.1 kg × (0.25 m)^2 = 0.61 kg-m^2

```
𝕀 = 1.8 * (0.25 )^2 + 1.8 * (0.25 )^2 + 3.1 * (0.25)^2 + 3.1 * (0.25)^2
```

```
0.6125
```

Part C: About which axis would it be harder to accelerate this array?
ANSWER: The one that has the largest moment of inertia is the hardest to accelerate and that is the vertical axis which has moment of inertia of 12.25 kg-m^2 Newton's 2md Law in angular form is $\tau=\mathbb{I}\alpha$ so for a given torque τ, the object with the larger moment of inertia $\mathbb{I}$ has the smaller acceleration α since $\alpha=\tau/\mathbb{I}$.

- **Problem #53**

A person stands, hands at his side, on a platform that is rotating at a rate of 1.3 rev/s If he raises his arms to a horizontal position as in figure 8-48 below, the speed of rotation decreases to 0.80 rev/s.
PART A: What does this happen?
ANSWER: Simplify the person arms by thinking of them replaced by two center-of-masses one for each arm each arm has a mass M. Initially the distance of the COM of each arm to the axis of rotation is $\mathbb{R}_0$ and after the person moves the arms into a horizontal position, the distance of the COM of each arm is $\mathbb{R}_f$ from the axis of rotation. From the diagram, it should be clear that $\mathbb{R}_f > \mathbb{R}_0$. The initial angular momentum $\mathbb{L}_0 = \mathbb{I}_0\ \omega_0$ where the initial angular momentum is ω_0 and $\mathbb{I}_0 = 2\,M\,\mathbb{R}_0{}^2$ is the initial moment of inertia when the arms are vertical or placed close to his side. The final angular momentum $\mathbb{L}_f = \mathbb{I}_f\ \omega_f$ where the final angular momentum is ω_f and $\mathbb{I}_f = 2\,M\,\mathbb{R}_f{}^2$ is the final moment of inertia when the arms are horizontal . Assume that angular momentum is conserved so that $\mathbb{L}_f = \mathbb{L}_0$ and this is true if there is no external torque acting on the system (and this is the case in this problem). Since $\mathbb{L}_f = \mathbb{L}_0$ it follows that $\mathbb{I}_f\ \omega_f = \mathbb{I}_0\ \omega_0$ and further $2\,M\,\mathbb{R}_f{}^2\,\omega_f = 2\,M\,\mathbb{R}_0{}^2\,\omega_0$. Solving for the final angular velocity $\omega_{\mathtt{f}}$ we obtain

$$\omega_{\mathtt{f}} = \frac{2\,M\,\mathbb{R}_0{}^2}{2\,M\,\mathbb{R}_f{}^2}\ \omega_0 = \frac{\mathbb{R}_0{}^2}{\mathbb{R}_f{}^2}\ \omega_0 = \frac{\mathbb{R}_0{}^2}{\mathbb{R}_f{}^2}\ \omega_0$$

Since $\mathbb{R}_f > \mathbb{R}_0$ it follows that $\omega_{\mathtt{f}} < \omega_0$ in other words, the angular velocity slows down as the persons arms are raised to a horizontal position.
PART B: By what factor has the moment of inertia changed?
ANSWER: Since $\mathbb{I}_f\ \omega_f = \mathbb{I}_0\ \omega_0$ by conservation of angular momentum, it follows

$$\frac{\mathbb{I}_f}{\mathbb{I}_0} = \frac{\omega_0}{\omega_f} = \frac{8.2\ \text{Rad/Sec}}{5.0\ \text{Rad/Sec}} = 1.6 \ \text{ so } \ \mathbb{I}_f = 1.6 * \mathbb{I}_0$$

since $\omega_0 = 1.3\ \frac{\text{Rev}}{\text{sec}} * 2\pi\ \frac{\text{Radians}}{\text{Rev}}$ =8.2 Rad/Sec. and $\omega_f = 0.8\ \frac{\text{Rev}}{\text{sec}} * 2\pi\ \frac{\text{Radians}}{\text{Rev}}$ =5.0 Rad/Sec So the final moment of inertia is roughly twice the initial moment of inertia and that is why the angular velocity decreases as the arms are raised to the horizontal position.

```
1.3 * 2 * π
```

```
8.16814
```

```
0.8 * 2 * π
```

```
5.02655
```

```
8.2 / 5.0
```

```
1.64
```

FIGURE 8–48
Problem 53.

- **Problem #55**

A figure skater can increase her spin rotation rate from an initial rate of 1.0 Rev every 2 seconds (which works out to 1/2 Rev/sec) to a final rate of 3.0 Rev/sec. If her initial moment of inertia was $\mathbb{I}_0$=4.6 kg-m^2 what is her final moment of inertia $\mathbb{I}_f$? How does she physically accomplish this change in rotation rate?

ANSWER: Conservation of angular momentum $\mathbb{L}_f = \mathbb{L}_0$ leads to $\mathbb{I}_f \ \omega_f = \mathbb{I}_0 \ \omega_0$. Solving for the final moment of inertia $\mathbb{I}_f$ you get

$$\mathbb{I}_f = \frac{\omega_0}{\omega_f} \mathbb{I}_0 = \frac{0.5 \text{ Rev/sec} * (2\,\pi \text{ Rad/Rev})}{3.0 \text{ Rev/sec} * (2\,\pi \text{ Rad/Rev})} * 4.6 \text{ kg-}m^2 = 0.77 \text{ kg-}m^2$$

So the final moment of inertia is smaller than the initial moment of inertia. This can be achieved by the skater moving her arms from a horizontal initial position to a final position with her arms to her side.

```
(0.5 / 3.0) * 4.6
```

```
0.766667
```