Lecture #2: Describing Motion-One Dimension

Define the parameters which describe motion of an object

1. Position of an object

2. Displacement of an object

3. Velocity of an object

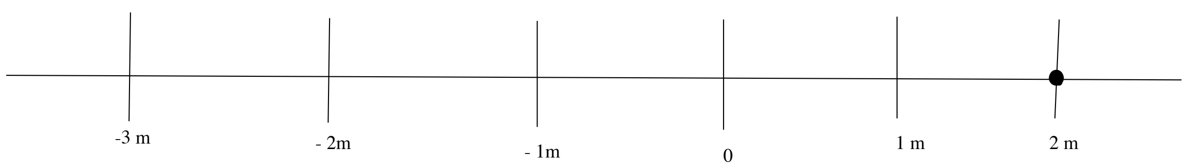
4. Speed of an object

5. Acceleration of an object

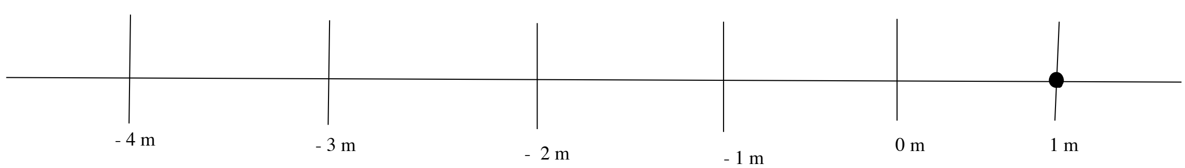
Position x of an Object:

The position of an object x is given in terms of a coordinate system (geometry)

Example: An object at position x = +2m for the coordinate system give below.



A different coordinate system will have a different value of x even though the object is at the same location.



Example above: X= 1 m for this second coordinate system which is a different X from before.
Note the object's position has not actually changed. What has changed is the coordinate system.

Frame of Reference:

1. Coordinate systems are called frames of reference in physics.

2. Coordinate systems can move in time.

3. The concept of a coordinate system is an imaginary construct. The actual location of the object is the same regardless how we describe its location.

- **EXAMPLE: A moving subway train car can have a coordinate system fixed inside the moving train car.**

4. Sometimes one coordinate system is better than another coordinate system. "Better" in this context means easier to use.

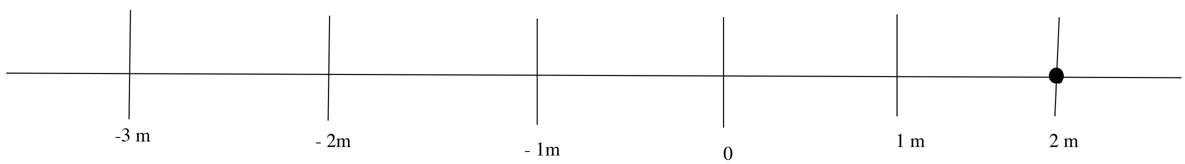
- **EXAMPLE: A ball rolling on the floor of a subway car**

The Displacement of an Object Δx

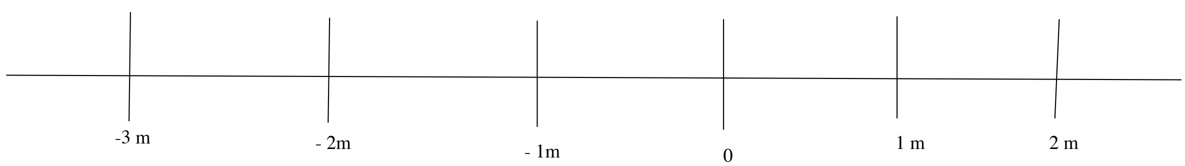
The displacement Δx of an object from the initial location point X_i to the final location point X_f is defined

$$\Delta X = X_f - X_i \tag{1}$$

Example: An object initially at position X_i = 2 m for the coordinate system given below



The object moves to the final position X_f = 3 m as indicated below



Notice the coordinate system or frame of reference has not moved in this example. It is the object that moved.

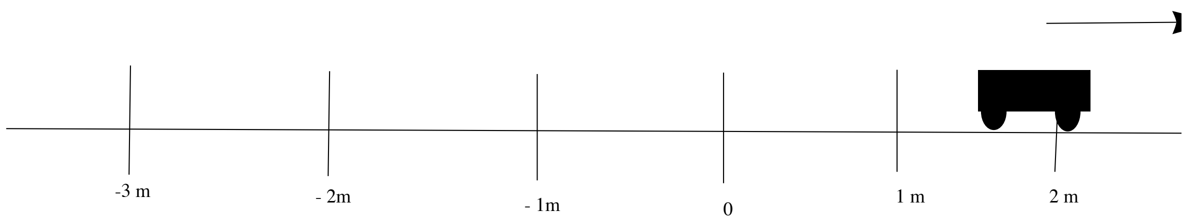
The displacement $\Delta X = X_f - X_i$ =3 m - 2 m = 1 m

The displacement ΔX is how far and in what direction the object has moved.

The displacement ΔX is positive if the object moves to the right. The displacement ΔX is negative if the object moves to the left.

The Speed and Velocity of an Object

Suppose an object is moving in the positive x direction as indicated in the diagram below:



Below is some position data collected at one second intervals for the motion of the object:

Time (sec)	0	1	2	3	4	5	6	7	8	9
Position (m)	0	2	4	6	8	10	12	14	16	18

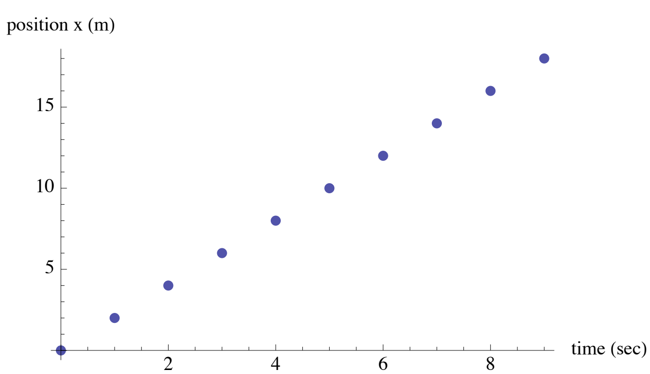
The data above is graphed below with the horizontal axis, time (sec) and the vertical axis, position (m):

data =

0	0
1	2
2	4
3	6
4	8
5	10
6	12
7	14
8	16
9	18

;

```
ListPlot[data, AxesLabel → {"time (sec)", "position x (m)"},
 AxesOrigin → {0, 0}, PlotStyle → PointSize[0.02]]
```



Interpolation of Data:

Even though data was collected a one second intervals, it is natural to interpolate the data and assume for example the when t=2.5 seconds, the position was 5 meters.

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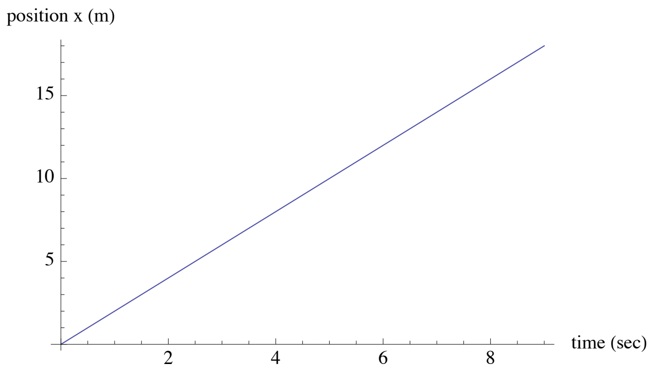
The Graph of the Function X=X[t] Representing the Motion:

Additionally, it is natural to connect the dots of the data above and obtain a straight line or linear function x=m t + b where m is the "slope" and b is the "intercept".

```
ListPlot[data, AxesLabel → {"time (sec)", "position x (m)"},
 AxesOrigin → {0, 0}, PlotStyle → PointSize[0.02], PlotJoined → True]
```



The Slope of the X=X[t] Graph: The Velocity

Since the graph passes through X=0 when t=0, the intercept b=0.
The "slope" m of a straight line is given by

$$m = \frac{X[t_2] - X[t_2]}{t_2 - t_1} \tag{2}$$

where 1 and 2 are any two points.

EXAMPLE: When t_2 =4 s the position is $X[t_2]$ = 8 m and when t_1 =3 s the position is $X[t_2]$ = 6 m so the slope is

$$m = \frac{8\,m - 6\,m}{4\,s - 3\,s} = 2\,\frac{m}{s} \tag{3}$$

The slope of the X versus t graph in physics has a special name: velocity

The graph of a straight line has a constant slope, so the velocity of the object above is a constant v=2 m/s.

The sign of the slope is important: The direction of travel

Example above: The slope and velocity is positive. v=+2 m/s The positive sign indicates the object is moving in the positive X direction.

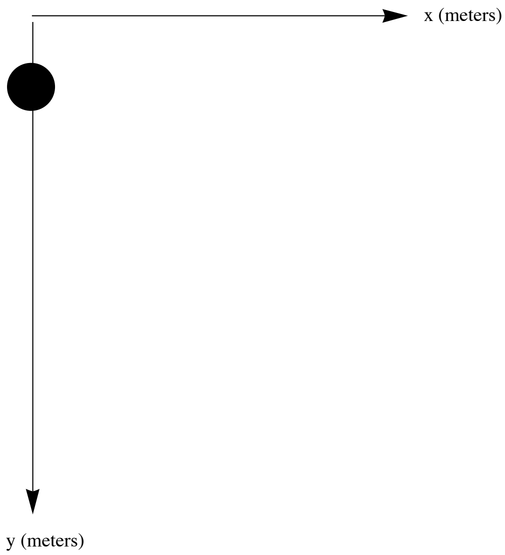
The Displacement: Notice the displacement is ΔX=2 m is constant for each pair of data points 1 sec apart is constant. There are motions where the displacment ΔX is not a constant (see below for example).

The Speed of an Object:

Sometimes the sign of the velocity is not important. For example, when driving a car you might only be interested in the reading of the "speedometer" to make sure you do not get a driving ticket. For this purpose if you are going 30 mph North, the direction "North" is unimportant and only the 30 mph is relevant. mph=miles per hour=miles/hour.

Non-Straight Line Data: Acceleration

EXAMPLE: Suppose an object is dropped from rest initially at t=0 s as indicated by the figure below:



There is no motion in the horizontal or x direction since the object is dropped from rest straight downward.

time (sec)	0	1	2	3	4	5	6
Position y (meters)	0	4.9	19.6	44.1	78.4	122.5	176.4

Notice the displacement Δy is NOT constant between pairs of data points.
Example: When going from 0 sec to 1 sec, the displacement Δy=(4.9 m - 0.0 m) = 4.9 m
When going from 1 sec to 2 sec, the displacement Δy=(19.6 m - 4.9 m) = 14.7 m
When going from 2 sec to 3 sec, the displacement Δy=(44.1 m - 19.6 m) = 24.5 m

So the displacement Δy is not constant and Δy is increasing with time. The is a characteristic of what is called accelerated motion.

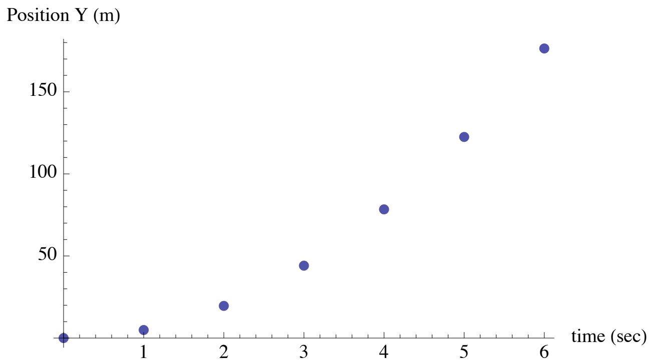
A Graph Y=Y[t] of the Data Above:

```
data =
```

0	0
1	4.9
2	19.6
3	44.1
4	78.4
5	122.5
6	176.4

```
;
```

```
ListPlot[data, AxesLabel → {"time (sec)", "Position Y (m)"}, PlotStyle → PointSize[0.02]]
```



The Average Velocity $\langle v \rangle$

Even when the velocity is not constant, the average velocity $\langle v \rangle$ can be defined between any two points. The average velocity $\langle v \rangle$ is defined by

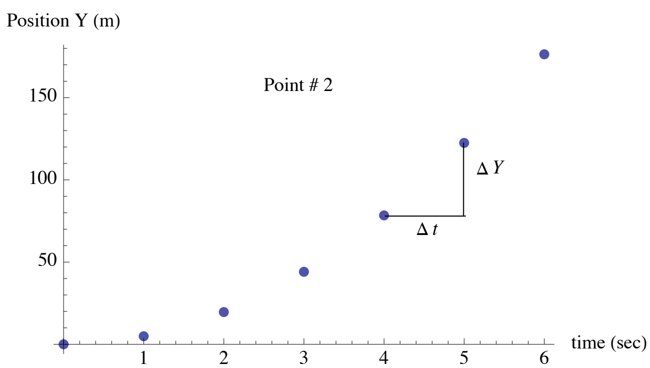
$$\langle v \rangle = \frac{Y[t_2] - Y[t_1]}{t_2 - t_1} = \frac{\Delta Y}{\Delta t} \tag{4}$$

EXAMPLE: Any two points may be taken in order to calculate the average velocity between those two points. For example, the average velocity going from 4 sec to 5 sec is computed thus

$$\langle v \rangle = \frac{122.5 - 78.4}{5 - 4}$$

44.1

so the result is 44.1 m/s



The average velocity $\langle v \rangle$ you compute depends on which pair of points you take. For example, the average velocity going from 3 sec to 4 sec is

$$\langle v \rangle = \frac{78.4 - 44.1}{4 - 3}$$

34.3

This average velocity $\langle v \rangle$ is smaller than when going from 4 sec to 5 sec and this make sense from the graph above since the slope is smaller in the second case.

More Data in the Interval 4 sec to 5 sec

One can imagine taking more data in the region going from 4 sec to 5 sec and one obtains

time (sec)	4.0	4.1	4.2	4.3	4.4	4.5	4.6	4.7	4.8	4.9	5.0
position y (m)	78.4	82.37	86.44	90.6	94.9	99.2	103.7	108.2	112.9	117.6	122.5

Computing the average velocity between pairs of points with one point always at 4 seconds we get

$$\langle v \rangle = \frac{122.5 - 78.4}{5 - 4}$$

44.1

$$\langle v \rangle = \frac{117.6 - 78.4}{4.9 - 4}$$

43.5556

$$\langle v \rangle = \frac{112.9 - 78.4}{4.8 - 4}$$

43.125

$$\langle v \rangle = \frac{108.2 - 78.4}{4.7 - 4}$$

42.5714

$$\langle v \rangle = \frac{103.7 - 78.4}{4.6 - 4}$$

42.1667

$$\langle v \rangle = \frac{99.2 - 78.4}{4.5 - 4}$$

41.6

$$\langle v \rangle = \frac{94.9 - 78.4}{4.4 - 4}$$

41.25

$$\langle v \rangle = \frac{90.6 - 78.4}{4.3 - 4}$$

40.6667

$$\langle v \rangle = \frac{86.44 - 78.4}{4.2 - 4}$$

40.2

$$\langle v \rangle = \frac{82.37 - 78.4}{4.1 - 4}$$

39.7

1. Observe the average velocity is smaller as point 2 moves closer to point 1.

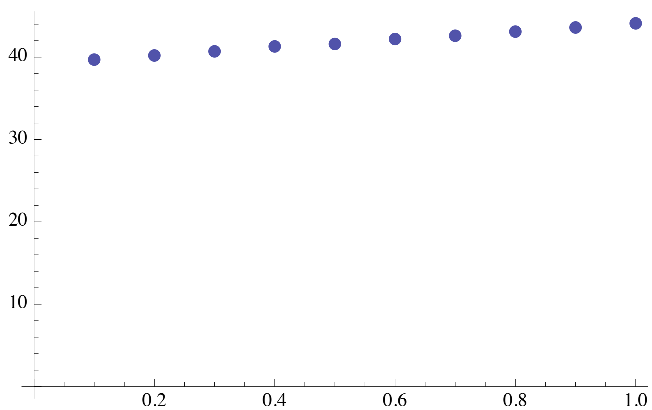
2. The average velocity appears to be approaching a limit of something like 40 m/s when point 2 is close to point 1.

vdata =

1	44.1
0.9	43.6
0.8	43.1
0.7	42.6
0.6	42.2
0.5	41.6
0.4	41.3
0.3	40.7
0.2	40.2
0.1	39.7

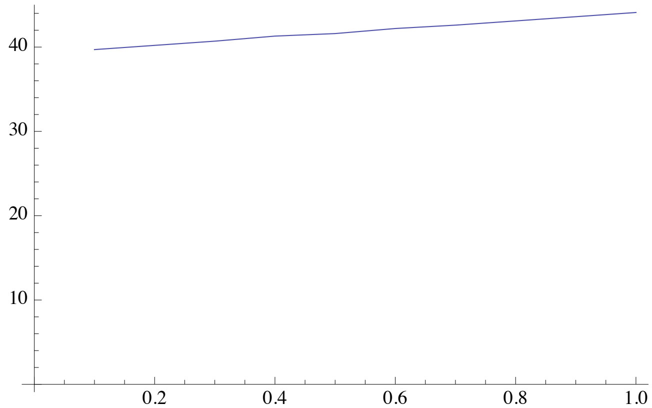
;

```
ListPlot[vdata, PlotStyle → PointSize[0.02], AxesOrigin → {0, 0}]
```



Extrapolation to Δt→0   The line crosses the vertical axis at about 39.2 m/s

```
ListPlot[vdata, PlotJoined → True, AxesOrigin → {0, 0}]
```



The Instantaneous Velocity v

It make some sense given the above example to define a velocity v at point 1 by the limit as $\Delta t \to 0$ as

$$v = \frac{Y[t_2] - Y[t_1]}{t_2 - t_1} \text{ as } t_2 \to t_1 \tag{5}$$

This velocity v is called the instantaneous velocity v. This is often written in the equivalent form

$$v = \underset{t_2 \to t_1}{\text{Limit}} \frac{Y[t_2] - Y[t_1]}{t_2 - t_1} \tag{6}$$

Since $\Delta Y = Y[t_2] - Y[t_1]$ by definition and $\Delta t = t_2 - t_1$ the definition of instantaneous velocity can be written most compactly as

$$v = \underset{\Delta t \to 0}{\text{Limit}} \frac{\Delta Y}{\Delta t} \tag{7}$$

The instantaneous velocity is about v=39.2 m/s at 4 sec in the example. The instantaneous velocity at 5 sec is different.

```
9.8 * 4
```

```
39.2
```

Graphical Picture: The instantaneous velocity is slope of the straight line tangent to the X versus t graph.

The Acceleration a

The graph of position X versus time t is often not a straight line. In these cases the instantaneous velocity is not constant. Suppose you take the following data for a falling object.

time (sec)	0	1	2	3	4	5	6
Position y (meters)	0	4.9	19.6	44.1	78.4	122.5	176.4
Instan velocity (m / s)	0	9.8	19.6	29.4	39.2	49	58.8

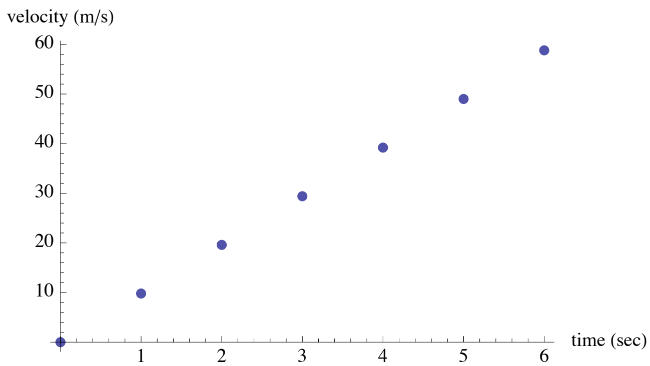
A graph of the instantaneous velocity v versus time t appears

vdata =

0	0
1	9.8
2	19.6
3	29.4
4	39.2
5	49
6	58.8

;

```
ListPlot[vdata, PlotStyle → PointSize[0.02], AxesLabel → {"time (sec)", "velocity (m/s)"}]
```



The graph of velocity versus time is a straight line.

The slope of the velocity versus time graph is constant.

The slope of the velocity versus time graph is called the acceleration.

Average Acceleration ⟨a⟩ defined

$$\langle a \rangle = \frac{v[t_2] - v[t_1]}{t_2 - t_1} \tag{8}$$

Constant Acceleration

Most of the time in this course the velocity versus time graph will be a straight line.

Since the slope is constant, the acceleration is constant in these cases.

There are a few cases (example, the mass on a spring) where the acceleration is not constant.

Instantaneous Acceleration

When the velocity versus time graph is not a straight line, the instantaneous acceleration is given by

$$a = \underset{t_2 \to t_1}{\text{Limit}} \frac{v[t_2] - v[t_1]}{t_2 - t_1} \tag{9}$$

What does NOT happen in nature:

You might think this process would go on for ever. For example, next we might graph the acceleration versus time.

Then we might assign the slope of the acceleration versus time graph a name. (There actually is no name for this in physics.)

It is surprising the nature does not work like this. While we can graph the acceleration versus time, this is not needed for the laws of physics.

Remarkably all that the laws of physics require is the position, velocity, and acceleration to describe the motion of an object.