Lecture #6: Newton's Laws of Motion

1. Galileo (1564-1642) studied the kinematic motion of objects and understood the concepts of velocity and acceleration. However, he did not use the concept of force so the cause of the motion was not explained. The telescope was invented and used for seeing objects in the distance on Earth. Galileo used the telescope to view the motion of moons of Jupiter. He understood this system was similar to the planets moving about the Sun.

2. Kepler studied (1571-1630) the motion of the planets about the Sun in detail and by observation he discovered three laws of planetary motion.

i) The motion of the planets about the Sun is elliptical with the Sun at one focus.

ii) Equal areas are swept out in equal times: planets when closer to the Sun move faster.

iii) The period T of the orbit of a planet is related to the "radius" a of the orbit by $T^2 = C\,a^3$ where C is a constant.

Kepler did not have much of an idea of the physics behind his laws.

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3. **Newton** (1642-1727) provided a physical understanding of the work of both Galileo and Kepler.

i) The physical understanding of Galileo's kinematics is given by **Newton's Three Laws of Motion**.

ii) The physical understanding of Kepler's laws of planetary motion uses Newton's three laws of motion together with **Newton's Law of Gravitation**.

These are examples of physical laws.

A Physical Law: A statement which describes in a concise way how nature behaves.

Usually a mathematical equation is involved.

Some Characteristics of Physical Law:

1. The physical law has the same form regardless of time and space.

Example: Newton's 2nd law of motion should be valid everywhere in our solar system or throughout the Universe.

2. Hierarchical Structure: Some ideas are more basic than others.

Physical Laws are the most basic level.

Theorems are obtained from the physical laws. (e.g. Homework)

Analogy: Euclid's Geometry (Five Postulates or Axioms)

3. A physical law cannot be proven correct. However a physical law can be proven wrong.

4. Physical laws usually have a domain where they are correct and also another domain where they are not correct.

Example: Newton's laws and Special Relativity Theory.

5. Scientific Revolutions Occur: An accepted paradign or model is used for a period of time and then it might be overthrown and replaced by a new paradign.

Newton' s Three Laws of Motion:

Newton's 1st Law of Motion: (1) An object in motion will remain in motion and (2) an object at rest will remain at rest unless acted upon by a force.

Mathematical Statement: If $\vec{F} = 0$ then $\vec{a} = 0$

where $\vec{F}$ is the total force acting on an object and $\vec{a}$ is the acceleration of the object.

Comments: Remember that the average acceleration is

defined

$\vec{a} = (\vec{v}_2 - \vec{v}_1)/t$. So when $\vec{a}$=0 this means $\vec{v}_2 = \vec{v}_1$ the velocity is unchanging.

If $\vec{v}_1$=0 then $\vec{v}_2$=0. If $\vec{v}_1 \neq 0$ then $\vec{v}_2 \neq 0$.

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Newton's 2nd Law of Motion: When a force $\vec{F}$ is applied to an object then it accelerates and the acceleration $\vec{a}$ is proportional to the force $\vec{F}$ with the mass m as the proportionality constant.

Mathematical Statement: $\vec{F}=m\vec{a}$.

Comments:

1. It might appear that the 1st law is a special case of the 2nd law and that the 1st law is redundant. That impression is wrong.

2. Newton's 2nd law is the one which you will find most useful in solving homework problems.

3. The 1st law is a statement of the **coordinate systems for which Newton's 2nd law holds**. These coordinate systems are called "inertial frames of reference".

4. Sometimes it is useful to use coordinate systems (frames of reference) for which Newton's 2nd law does NOT hold. These are called "non-inertial frames of reference".

Newton's 3rd Law of Motion: For every action (force) there is an equal and opposite reaction (force).

Mathematical Statement: Suppose there are two objects labeled 1 and 2 and the force exerted by object 2 on object 1 is called $\vec{F}_{2\,\text{on}\,1}$. Then the force on object 1 by object 2 that is $\vec{F}_{1\,\text{on}\,2}$ is given by the relationship

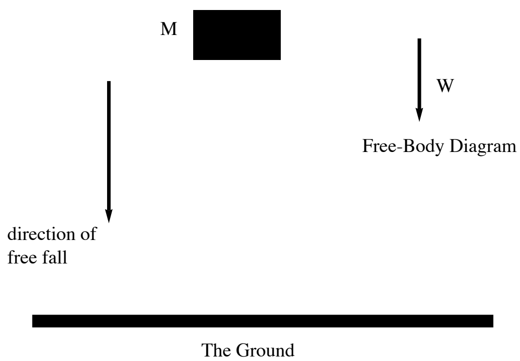
$\vec{F}_{2\,\text{on}\,1} = -\vec{F}_{1\,\text{on}\,2}$.

Comments:

1. Newton's 3rd law holds for Newton's Law of Gravitation which gives the force between say an apple and the Earth. It is perhaps surprising that the force of the Earth on an apple is the same magnitude as the force of the apple on the Earth!

2. Newton's 3rd law does NOT hold for magnetic force since the magnetic force depends upon how fast the object is moving not just its location.

1. Example of Using Newton's 2nd Law of Motion: The free fall of a mass M acted upon by the force of gravity W downward toward the Earth



A "Free-Body Diagram" is shown to the right of the falling mass. The "Free-Body Diagram" does not show the falling body (hence the diagram is free of the body) and only the force(s) acting on appear. In this case there is only one force W. Newton's 2nd law $\vec{F} = m\,\vec{a}$ for this problem is

$W = M\,g$

i) The motion is only in one dimension so the vector symbol is not necessary.

ii) The mass M is measured in kilograms with a balance.

iii) The acceleration of all masses is the same $a=g=9.8\ m/s^2$ at sea level.

iv) The force of gravity has the special symbol W and has unit of Newtons or Nt. $1\ Nt \equiv kg \times m/s^2$. The Newton is a ***derived*** unit.

Question: Suppose you have a 3 kg mass. What is the force of gravity that acts on that mass?

Answer: $W = M g = 3 \text{ kg} \times 9.8 \text{ m/s}^2 = 29.4 \text{ Nt}$

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9.8 * 3
29.4
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Question: What would happen to Newton's 2nd law for the mass M taken into space and the mass M is very far from the Earth?

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Answer:

i) The mass is not acted on by any force so W=0 Nt which means the mass has zero weight or is "weight-less".

ii) The amount of the mass in kilograms stays the same since the inertia (mass) or "resistance" to changes in motion stays the same.

iii) Suppose the acceleration of the mass M in outer space is a=0. This means that if the mass has zero velocity, the velocity of the mass remains zero.

iv) So both F and a are zero in Newton's 2nd law F= M a while only the mass retains the value it had on Earth.

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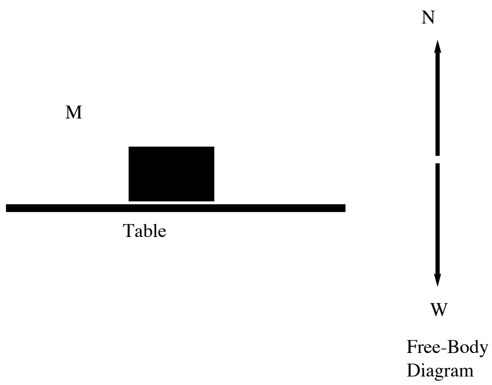
2. Example of Newton's 2nd Law: A object having mass M is at rest on top of a table.



There two force acting on the mass M as indicated in the free-body diagram.

i) One **force is gravity** W and this force is the same as in the previous example since nothing has changed with regard to gravity. It is NOT a good idea to think of W as the weight of the mass because the only time W equals the weight is the example #1 above.

ii) There also is the force N of the table on the mass M. N is in the upward direction. (N is perpendicular to the table surface so N is in the normal direction hence the use of the symbol "N".) If a bathroom scale were placed between the mass M and the table, then the scale would read the force N in Newtons Nt. So N is what one normally thinks of as the weight of the object. (We use the symbol Nt for Newtons so to not confuse it with N.)

iii) The acceleration of the mass is zero a=0 since it is assumed the mass is initially at rest.

iv) Newton's 2nd law F=Ma becomes

$N-W=N\times 0$

and this means N-W=0 or N=W. So the normal force equals the force of gravity W.

v) If the table were magically removed, the mass M would fall the same way as in example #1. So the relationship W=Mg holds in example #2 as well. (Removing the table has no effect on the force of gravity W or the acceleration g of the mass.)

vi) N = W = Mg if we use the same mass M= 3 kg as before then N= W = M g = 3 kg × 9.8 m/s^2 = 29.4 Nt is the Normal force.

NOTE: We can use the relationship W=Mg ***even though the mass is at rest on the table top and the acceleration a=0 (and NOT g=9.8 m/s^2).***

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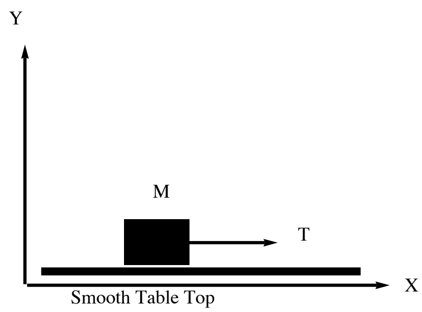
3. **Example of Using Newton's 2nd Law of Motion**:

Consider a mass M being pulled by an attached rope as indicated below.

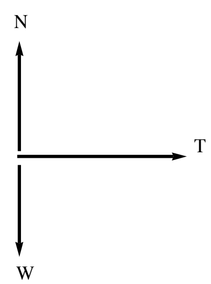


i) The X, Y coordinate system (or frame of reference) used for this problem is indicated. The frame of reference is not moving with respect to the table top and the motion of the mass M is only in the x direction.

ii) There is no friction between the mass M and the table top for now but later on friction will be included.

iii) The force in the rope is called a Tension and is labeled T.

iv) The free-body diagram for this problem appears below:



v) The force of gravity or weight W and the normal force N were discussed before. Only the tension T is new.

v) Newton's 2nd law appears in vector form as $\vec{F} = M\vec{a}$ but it actually stands for two equations: $F_x = M a_x$ and $F_y = M a_y$.

vi) F_x is the total force in the x-direction and in this problem F_x=T. So applying Newton's 2nd law to the motion in the x-direction you get $T = M a_x$.

A Numerical Example: Suppose M=5 kg and T=2 Nt then the acceleration in the x-direction is a_x=T/M=$\frac{2\,N}{5\,\text{Kg}} = 0.4\,\text{m}\big/\text{s}^2$.

v) F_y is the total force in the y-direction and in this example F_y=N-W. There is no movement in the y-direction so a_y=0 and Newton's 2nd law becomes N-W = $F_y = M a_y$=0 so N-W=0 or N=W=Mg.

A Numerical Example: Since M=5 kg the force of gravity is W=5 kg × 9.8 m/s^2=49 Nt so the normal force is N=49 Nt is the upward normal force of the table on the mass M.

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5. * 9.8
49.
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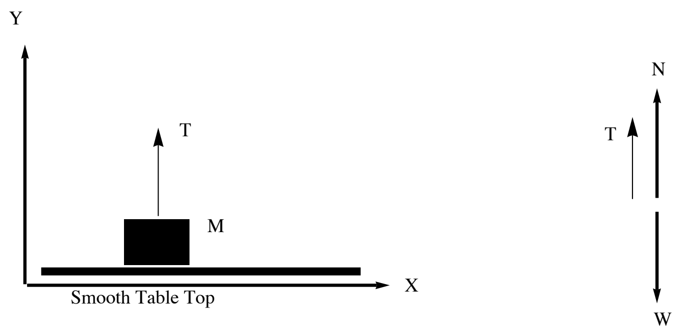
4. **Example of Newton's 2nd Law: Pull up on a Mass by a Rope**

Suppose you attach the rope on top of the mass M as indicated by the diagram below



The free-body diagram is on the right and there are now three forces in the y-direction (and none in the x-direction). The total force acting on the mass M is $F_y = N + T - W$ and Newton's 2nd Law appears

$N + T - W = M\ a_y$

There three possibilities:

1. $N + T - W > 0$ in which case there is a vertical acceleration (the mass rises upward) given by $a_y = F_y/M$ or since $F_y = N + T - W$

a_y= (N + T - W)/M

2. N + T - W < 0 in which case the acceleration is negative which is impossible since this means the mass goes downward through the table top.

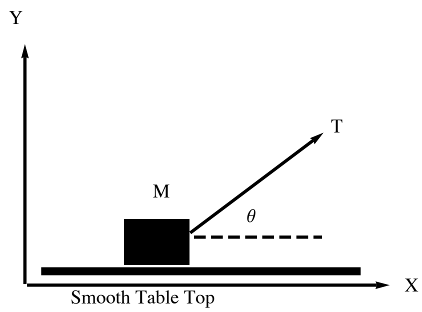
3. N + T - W = 0 in which case the acceleration is zero. The normal force N = W - T is less than the weight W of the mass since the rope reduces the normal force.

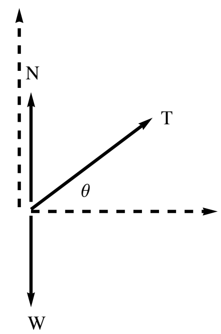
A Numerical Example: Using the same mass M as before we compute

M=5 kg W=5 kg × 9.8 m/s^2=49 Nt as the weight W. Suppose the tension in the rope is T=20 Nt, then the normal force is N = 49 Nt - 20 Nt = 29 Nt. The normal force N is what a scale would read that is placed between the mass M and the table so N is what is normally called the "weight" of an object. The scale would NOT read W = 49 Nt. W is the force of gravity on M.

5. Example of Newton's 2nd Law: The Tension at an Angle θ

The diagram below indicates the physical situation in this case





and the free-body diagram appears to the side.

i) Newton's 2nd law appears as before in component form

$F_x = M\, a_x$ and $F_y = M\, a_y$.

ii) The tension has two components now:

T Cos[θ] is the x-component and

T Sin[θ] is the y-component.

iii) F_x = T Cos[θ] is the total force in the x-direction.

F_y = T Sin[θ]+N-W is the total force in the x-direction.

There are now three possibilities:

1. T Sin[θ]+N-W > 0 in which case the tension in the rope is enough to cause the mass to rise upward with a positive acceleration.

2. T Sin[θ]+N-W < 0 in which case the acceleration is negative which is impossible since that would mean the mass goes downward through the table.

3. T Sin[θ]+N-W = 0 in which case there is zero acceleration in the y-direction. The normal force N can be obtained thus

N= W - T Sin[θ]

The acceleration in the x-direction in this case is given by

$a_x = \frac{F_x}{M} = \frac{T}{M}$ Cos[θ]

Questions:

1. Is the normal force N greater or less than in problem #4? Explain your answer.

2. Is the acceleration greater or less than in problem #4? Explain your answer.

A Numerical Example: Use the same values mass M, force fo gravity W, and the tension T in the rope as in the previous example. M=5 kg, W=5 kg × 9.8 m/s^2=49 Nt, T= 20 Nt and chose θ=30°. The normal force N = 49 Nt - 20 Nt × Sin[30°] or N=39 Nt.

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n = 49. - 20 * Sin[30 °]
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39.
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The acceleration a_x= $\frac{20.}{5.}$× Cos[30°]=3.5 Nt

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ax = 20./5 * Cos[30 °]
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3.4641
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