# Work and Conservation of Energy 

## Topics Covered:

1. The definition of work in physics.
2. The concept of potential energy
3. The concept of kinetic energy
4. Conservation of Energy

## General Remarks:

Two Equivalent Description of Motion:

1. Newton's laws of motion especially F=ma

Forces, mass, velocity, acceleration etc.
2. Conservation Principles

Energy
Momentum (linear and angular)
Notes:

1. One description might be easier to apply in a given physical situation.
2. The laws of energy and momentum conservation are more general principles than Newton's Laws. e.g. Thermodynamics (no forces)

## The Definition of Work W

The work done by a force $\vec{F}$ in moving an object a distance $s$ is given by
$W=F s$
where is was assumed that $\vec{F}$ is in the same direction as the displacement $\vec{s}$.
Example: Suppose $\vec{F}$ acts on a box as indicated below and the box moves a distance $\overrightarrow{\mathrm{s}}$ :


## Notes:

1. Whether or not there is a friction force is not relevant to the definition of the work done by the force $\vec{F}$. However, friction is important as the what the final state of motion of the box of course. 2. The symbol W is used for both work and the force of gravity $\mathrm{W}=\mathrm{mg}$. This is unfortunate but common usage. Work and the force of gravity do not even have the same units. Usually the context will tell you whether work of gravitational force is meant.
2. Observation: If the force $\vec{F}$ does not produce a displacement so that $\vec{s}=0$ while $\vec{F}$ acts on the mass, then the work done is zero $W=0$. This is different from the popular notion of work.
3. It is assumed that the force $\vec{F}$ is constant during the displacement of the object. If it is not constant, the definition of work is modified in a simple manner (see below).
4. Units of Work: $\mathrm{W}=\mathrm{F}$ s S.I. or M.K.S. System: Joule $=$ Newton $\times$ meter of $\mathrm{J}=\mathrm{Nt} \times \mathrm{m}$

## The Case where $\vec{F}$ is NOT in the same direction as the Displacement $\overrightarrow{\mathrm{s}}$ :

When $\vec{F}$ is not in the same direction as $\vec{s}$ then only the component of force $\vec{F}$ in the same direction as $\stackrel{\rightharpoonup}{s}$ contributes to the work done. The component of the force $\vec{F}$ not in the direction of $\vec{s}$ does not produce any motion and hence no work is produced by this component of force $\vec{F}$. This should seem reasonable given what was said above about work.

Example: Consider a force $\vec{F}$ applied through a rope attached to a box and the direction of the force $\overrightarrow{\mathrm{F}}$ is upward at an angle $\theta$ with respect to the horizontal.


The component of the force $\vec{F}$ in the direction of the displacement $\vec{s}$ is given by $F \operatorname{Cos}[\theta] s$ where more generally $\theta$ is the angle between $\vec{F}$ and $\overrightarrow{\mathrm{s}}$. Next we will calculate the work done in various situations.

## The Work Done by Lifting a Mass ma height h:

Consider the situation indicated by the drawing below. The external force $\vec{F}$ pulls on a rope attached to the top of the mass $m$ and in this case, the box moves in the upward direction.


The displacement $\overrightarrow{\mathrm{S}}$ is in the same direction as the external force $\overrightarrow{\mathrm{F}}_{\text {ext }}$ and conventionally $h=|\overrightarrow{\mathrm{S}}|$ is called h the height the mass is raised upward.

$$
\mathrm{W}=F_{\mathrm{ext}} h
$$

is the work done by the force $F_{\text {ext }}$ in moving the mass $m$ upward a distance $h$.
While the mass is being moved upward, the force of gravity is also acting on the mass.
What is the work done by gravity in this process?

## The Work Done by Gravity in the Process where a Mass is Raised a Height h:

The force of gravity $F_{\text {gravity }}=\mathrm{mg}$ is downward in the opposite direction to the displacement $h$ which is upward.


So the angle between $F_{\text {gravity }}=\mathrm{mg}$ and h is $\theta=180^{\circ}$ and the work done by the force of gravity is $\mathrm{W}=F_{\text {gravity }} \mathrm{h} \operatorname{Cos}\left[180^{\circ}\right]=\operatorname{Meg} \mathrm{h}(-1)$ since the $\operatorname{Cos}[180]=-1$.
$W_{\text {gravity }}=-m g h$
is the work done $B Y$ the gravity force $F_{\text {gravity }}=m g$ in the process of moving the mass m upward a distance $h$.

NUMERICAL EXAMPLE: Suppose $\mathrm{m}=3 \mathrm{~kg}$ and $\mathrm{h}=5$ meter then $\mathrm{W}_{\text {gravity }}=-3 \mathrm{~kg} \times 9.8 \mathrm{~m} / \mathrm{s} \times 5 \mathrm{~m}$ which works out to $\mathrm{W}_{\text {gravity }}=-150 \mathrm{~J}$ where $\mathrm{J}=$ Joule is a unit of energy. One Joule $=\mathrm{Nt} \times \mathrm{m}$ WHAT IS THE MEANING OF THIS MINUS SIGN? Answer: Instead of work being done by gravity, work is instead done against or on gravity in other words $W_{\text {on }}=-W_{b y}$ PROBLEM: If the external force is just equal and opposite to the force of gravity, then the net force on the mass is zero and the mass will NOT MOVE the height $h$ (if it is not moving initially). What do we do about this?

## The amount of work done depends upon the specific process by which the mass moves.

i) If the external force $F$ is just equal and opposite the force of gravity mg , then the mass will not move. But somehow the mass is raised a height $h$. How is this done?
ii) Imagine that the external force $F$ is slightly greater than the force of gravity to get the mass moving upward.
iii) Once the mass is moving, the external force is made just equal and opposite mg the force of gravity. The mass moves at constant speed for the major part of the trip a distance h . iv) When the mass gets close to having moved the distance h upward, the external force is reduced a little less than mg so the mass comes to rest.
v) For most of the trip from the initial position to the final position, the mass has a constant, small velocity (we can make this velocity as small as we wish).
vi) We are justified in taking $F_{\text {ext }}=m g$ and the work done by $F_{\text {ext }}$ is $W_{\text {ext }}=M g h$ for the bulk of the trip from the initial position to the final position. Notice this is the negative of the work done by gravity which is $\mathrm{W}_{\text {gravity }}=-\mathrm{Mgh}$

NUMERICAL EXAMPLE: Take the same example as before where mass $\mathrm{m}=3 \mathrm{~kg}$ is raised a height $\mathrm{h}=5$ meter then $\mathrm{W}_{\text {ext }}==3 \mathrm{~kg} \times 9.8 \mathrm{~m} / \mathrm{s} \times 5 \mathrm{~m}$ which works out to $\mathrm{W}_{\text {ext }}=150 \mathrm{~J}$. Recall the work done by gravity was the negative of this, that is $W_{\text {gravity }}=-150 \mathrm{~J}$.

## Gravitational Potential Energy P.E.

1. Potential Energy is one of several kinds of energy discussed in this course.
i) Potential Energy P.E. is "energy of position".
ii) Potential Energy is the amount of "energy available to do work".
2. Right now the focus is on a kind of potential energy called gravitational potential energy.

ASIDE: There are other kinds of potential energy. Example: electrical potential energy which you will study next semester.

## Definition of the Change in Potential Energy:

Suppose $\mathrm{W}_{\text {ext }}$ is the amount of work done by $\mathrm{F}_{\text {ext }}$ which is necessary to move a mass from an original vertical position to the final position in the presence of the gravity force. The change in Potential Energy or $\Delta \mathrm{PE}=\mathrm{PE}_{f}-\mathrm{PE}_{o}$ in moving a mass from the original position to the final position is defined as
$\Delta \mathrm{PE}=\mathrm{W}_{\text {ext }}$ or equivalently $\Delta \mathrm{PE}=-\mathrm{W}_{\text {gravity }}$.

## Application to the Gravitational Force:

Suppose you have a mass which is raised a height $h$ which again appears as below:


We got before that the work done by gravity in this process is $W_{\text {gravity }}=-m g h$. So the change in gravitational potential energy in this process is $\Delta \mathrm{PE}=-(-\mathrm{mgh})=\mathrm{mgh}$ or more simply $\Delta \mathrm{PE}=\mathrm{mgh}$.

Reason for the Minus Sign: Notice the two minus signs cancel to give a positive $\Delta \mathrm{PE}=\mathrm{PE}_{f}-\mathrm{PE}_{o}>0$ which means the final potential energy $\mathrm{PE}_{f}$ is greater than the initial potential energy $\mathrm{PE}_{o}$. This agrees with your intuition that a mass raised a height h has a greater Potential Energy than before it was raised.

## IMPORTANT NOTES:

1. Implicit in this definition of $\Delta \mathrm{P} . \mathrm{E}$. is the condition that the initial and final velocities are zero and that the mass is moved as slowly as possible from the initial position to the final position.
2. If you do not follow this process and say for example, suppose the mass is moving at the final position then more work is done than $\mathrm{W}=\mathrm{mgh}$ in the process and the extra work will appear in what is called Kinetic Energy or "energy of motion".
3. Notice the change of potential energy $\Delta P$.E. was defined and NOT the Potential

Energy P.E. itself. This is important as will be seen later.
4. Potential Energy Units: Same as work (Joules)

## The Case Where the Mass $m$ is Lowered a Distance h:

Suppose the mass m is returned to the initial position as indicated by the drawing below.


1. Notice that the force $\vec{F}$ is in the opposite direction as the displacement $\overrightarrow{\mathrm{s}}$ so the angle $\theta=180^{\circ}$ between $\vec{F}$ and $\vec{s}$. Since $\operatorname{Cos}\left[180^{\circ}\right]=-1$ the work done is $W=-F h$ with $h=|\vec{s}|$. Since $F_{\text {ext }}=m g$ upward, it follows that the work done in the above process is
$W_{\text {ext }}=-m g h$
The minus sign in the above equation is interpreted as work done by the upward lifting force $F$ is negative, that is work is done on the lifting force (instead of work done by the lifting force in the previous case where $\mathrm{W}_{\text {ext }}=+$ mgh with a + sign).
2. The work done by gravity is positive now since the force of gravity is in the same direction as h so $\mathrm{W}_{\text {gravity }}=\mathrm{mgh}$
3) The change in gravitational Potential Energy is $\mathrm{PE}=-\mathrm{W}_{\text {gravity }}=-\mathrm{mgh}$ is negative in this processes. This means there was a loss of PE when the mass was lowered a distance $h$.

## Some Properties of Gravitational Potential Energy

Property \#1: The reason for the minus sign in the definition $\Delta$ P.E. $=-m g h$ can be seen if you write $\Delta \mathrm{PE}=\mathrm{PE}_{f}-\mathrm{PE}_{i}$ then for gravity $\mathrm{PE}_{f}=\mathrm{PE}_{i}-\mathrm{mgh}$

Property \#2: The zero level for computing the change in potential energy is arbitrary. (This is similar to the idea that the coordinate system you use to solve a problem with Newton's 2nd Law is arbitrary and is your choice.)

Example: Suppose you consider raising a mass off a table vertically a height h as pictured below.


Calculate the change in Potential Energy $\triangle \mathrm{PE}=\mathrm{PE}_{f}-\mathrm{PE}_{i}$ of going from the initial position to the final position in two different methods:

## Method \#1: Using the table top as the reference level (or origin of coordinates):

$\Delta \mathrm{PE}=\mathrm{mgh}$
Comments: Since the mass is already at the reference level, no work is done to get it there and to move the mass upward a height $h,-m g h$ work was done by gravity so $\Delta P E=-W=m g h$.

Way \#2: The ground as the reference level.
The mass starts at a height $\ell$ above the ground reference level and since work $\mathrm{W}=-\mathrm{mg} \ell$ is done to get it there so $\mathrm{PE}_{o}=\mathrm{mg} \ell$ is the potential energy at the initial position with respect to the ground reference level. Moving the mass upward a height h above the table would require $\mathrm{W}=-\mathrm{mg}(\ell+\mathrm{h})$ starting from the reference level so $\mathrm{PE}_{f}=\mathrm{mg}(\ell+\mathrm{h})$. The change in potential energy is $\Delta \mathrm{PE}=\mathrm{PE}_{f}-\mathrm{PE}_{o}=\mathrm{mg}(\ell+\mathrm{h})-\mathrm{mg} \ell=\mathrm{mgh}$

IMPORTANT OBSERVATION: $\triangle \mathrm{PE}=\mathrm{mgh}$ in both METHODS So the change in potential energy is independent of the choice of reference level. You can choose the reference to make solving a problem as easy as possible.

Property \#3 of Gravitational Potential Energy: The change in gravitational potential energy in moving from point $A$ to point $B$ is independent of the path you take to get from $A$ to $B$.

EXAMPLE: Suppose the initial point $A$ and final point $B$ are as indicated below.


Use the ground as the reference level for all the calculations of $\triangle P E$.
PATH \#1: $\Delta \mathrm{PE}=\mathrm{mgh}+0=\mathrm{mgh}$ (the work done in the horizontal piece is zero because the force is perpendicular to the displacement.)

PATH \#2: $\Delta \mathrm{PE}=0+\mathrm{mgh}=\mathrm{mgh}$
PATH \#3: $\triangle \mathrm{PE}=\mathrm{mgh}$
COMMENTS: The path \#3 can be broken up into small vertical pieces plus horizontal pieces. The work done in the horizontal pieces is zero because the force of gravity is perpendicular to the displacement. The work done in the vertical pieces is added together to get mgh .

ARBITRARY PATH: Any arbitrary path from A to B can be approximated by small vertical and horizontal pieces so the net work is mgh .

CONSERVATIVE FORCE: The work done by gravity is independent of the path taken. Gravity is an example of a "conservative force" because the work done is independent of path.

EXAMPLE: Non-conservative force is friction.

## Kinetic Energy: Energy of Motion

EXAMPLE: Suppose a force F pulls a rope attached to a mass M as indicated below:


The work done by the force F is $\mathrm{W}=\mathrm{FS}$. Recall the kinematic equation
$\mathrm{V}_{f}{ }^{2}=\mathrm{V}_{0}{ }^{2}+2 \mathrm{aS}$
and remember Newton's 2nd Law in the form
$\mathrm{a}=\frac{\mathrm{F}}{\mathrm{m}}$
so you get after eliminating the acceleration a
$\mathrm{V}_{f}{ }^{2}=\mathrm{V}_{\mathrm{o}}{ }^{2}+2 \frac{\mathrm{~F}}{\mathrm{M}} \mathrm{S}$
Rearrange a little bit an obtain
$\frac{1}{2} M V_{f}{ }^{2}=\frac{1}{2} M V_{o}{ }^{2}+F S$
Recall the definition of work W = F S and finally get
$\frac{1}{2} M V_{f}{ }^{2}=\frac{1}{2} M V_{o}{ }^{2}+W$
This equation is called the Work-Energy Theorem.
The Kinetic Energy is defined $K E=\frac{1}{2} M V^{2}$ and using this definition the Work-Energy Theorem can be written
$\mathrm{W}=\mathrm{KE}_{f}-\mathrm{KE}_{o}$
Work-Energy Theorem: If an amount of work W is done on a mass M then W is equal to the change in Kinetic Energy.

COMMENTS:

1. The Work-Energy Theorem applies only if the Potential Energy PE of the mass M does not change. This means the mass $M$ was not raised or lowered.
2. If the Potential Energy does change, then Conservation of Energy applies as will be seen shortly.
3. The above example assumes there is no friction force. If there is friction then the work against friction must be included in the Work-Energy Theorem.

## "Proof" of Conservation of Energy: A Falling Mass

Suppose you have a mass M a height h above the above as indicated below:


This time things are a little different than when we discussed this situation before. This time release the mass $M$ from rest and let the mass fall on its own due to the gravitational force. There is no other force than gravity acting on M. Just before the mass hits the ground the downward velocity is $\mathrm{V}_{f}$. So this situation is different since the mass is moving substantially at the final position (just above the ground). The kinematic relationship holds
$\mathrm{V}_{f}{ }^{2}=\mathrm{V}_{0}{ }^{2}+2 \mathrm{a} \Delta \mathrm{h}$
where $\Delta \mathrm{h}=\mathrm{h}_{f}-\mathrm{h}_{0}$.
Newton's 2nd Law gives the acceleration of the mass $a=F / M$ where $F$ is the force of gravity so the kinematic relationship above becomes
$\mathrm{V}_{f}{ }^{2}=\mathrm{V}_{0}{ }^{2}+2 \frac{F}{M} \Delta \mathrm{~h}$
After multiplication by $\mathrm{M} / 2$ the above equation becomes
$\frac{1}{2} m V_{f}{ }^{2}=\frac{1}{2} m V_{0}{ }^{2}+F \Delta h$
The last term of the above equation is $\mathrm{W}=\mathrm{F} \Delta \mathrm{h}$ and this is the work done by gravity when the mass goes from $\mathrm{h}_{\mathrm{o}}$ to $\mathrm{h}_{\mathrm{f}}$. Recall the discussion above associated with the change in gravitational Potential Energy with (the negative) of the work done by gravity that is
$W=F \Delta h=-\Delta P E$
where as usual $\Delta P E=P E_{f}-P E_{0}$. Combining the above yields
$\frac{1}{2} m V_{f}{ }^{2}=\frac{1}{2} m V_{0}{ }^{2}-\Delta \mathrm{PE}$
in other words
$K E_{f}=K E_{o}-\left(P E_{f}-P E_{o}\right)$
since $\mathrm{KE}_{\mathrm{f}}=\frac{1}{2} \mathrm{~m} \mathrm{~V}_{f}{ }^{2}$ and $\mathrm{KE}_{o}=\frac{1}{2} \mathrm{mV} \mathrm{V}_{o}{ }^{2}$. One further rearrangement yields
$K E_{o}+P E_{o}=K E_{f}+P E_{f}$
This is called the Conservation of Mechanical Energy where the Total Mechanical Energy is $\mathrm{TE}=\mathrm{KE}+\mathrm{PE}$.

## COMMENTS:

1. The above situation has the mass $M$ is moving at the final position (and possibly at the initial
position if we do not release the mass from rest). None-the-less we will take the change in Potential Energy to be the same as when the mass was slowly lowered.
2. Conservation of Mechanical Energy for the above example follows from Newton's 2nd Law and the kinematic relation. However, we will assume that Conservation of Energy holds in other situations as well provided there is no friction or heat loss (or gain) to the mass.
3. It should be kept in mind that a law of nature (like Conservation of Energy) cannot be derived. A law of nature can be a consequence of another law of nature (in this case Newton's 2nd law). Which of the laws (Newton's 2nd or Conservation of Energy) is more fundamental is sometimes a matter of taste. In the future, we will assume Conservation of Energy holds in order to solve other physics problems even though we will have NOT proved Conservation of Energy energy is in fact true and this is an OK procedure.
4. Kinetic Energy and Potential Energy are examples of Mechanical Energy. A non-mechanical form of energy is Heat Energy for example. The Heat energy is associated with the random motion of molecules of a gas. If a gas acquires Heat Energy, the random motion of the molecules increases. On a microscopic level, Heat Energy is mechanical in the sense it is due to the Kinetic Energy of the molecules making up the gas. However, the behavior of Heat Energy is governed by the Laws of Thermodynamics which CANNOT be derived from Newton's Laws of Motion. Additional probability assumptions need to be made. This will be discussed more toward the end of this course.

## EXAMPLE: A Mass Sliding Down and Inclined Plane (no friction):

Consider a block having mass M sliding down an inclined plane as indicated by the diagram below. The angle $\theta$ of the inclined plane with the horizontal is also shown.


Use the bottom of the inclined plane as the Reference Level for calculating potential energy. The initial Potential Energy is $\mathrm{PE}_{o}=\mathrm{Mgh}$ since the mass M is a height h above the reference level initially. If you are given the distance down the plane $R$ when the mass has gotten to the final position, then the final position height is $\mathrm{R} \operatorname{Sin}[\theta]$ and the final Potential Energy is $\mathrm{PE}_{f}=\mathrm{MgR} \operatorname{Sin}[\theta]$

Suppose the mass is released from rest so that the initial velocity $\mathrm{V}_{0}=0$ and the initial Kinetic

Energy is $\mathrm{KE}_{o}=\frac{1}{2} M \mathrm{~V}_{o}{ }^{2}=0$. We suppose the final velocity $\mathrm{V}_{f}$ is what we want to calculate but if we knew $\mathrm{V}_{f}$, then the final Kinetic Energy is $\mathrm{KE}_{f}=\frac{1}{2} M \mathrm{~V}_{f}{ }^{2}$. Conservation of mechanical energy $\mathrm{KE}_{o}+\mathrm{PE}_{o}=\mathrm{KE}_{f}+\mathrm{PE}_{f}$ applied to this problem yields $0+M g h=\frac{1}{2} M \mathrm{~V}_{f}{ }^{2}+M g R \operatorname{Sin}[\theta]$

The mass M cancels out and solving for $\mathrm{V}_{f}$ we get

$$
\mathrm{V}_{f}=\sqrt{2 g(h-R \operatorname{Sin}[\theta])}
$$

Comments:

1. You would get the same result if you used Newton's 2nd Law of Motion and you should convince yourself of that. Also, you will realize that the Newton's Law method involves a bit more calculations than Conservation of Energy.
2. By the way, since the mass $M$ does NOT appear in the answer for the final velocity $\mathrm{V}_{f}$, two blocks with two different masses will reach the final position at the same time. This is the same conclusion that we obtained from the Galileo experiment in which the two masses fall directly downward (and there is not inclined plane). Would you get the same result if there were friction? 3. Compare the time it takes a mass to slide down the inclined plane with the time it takes a mass to fall straight downward and the final vertical position is the same in both cases. Convince yourself that the velocities of the two masses would be the same. However, the mass sliding down the inclined plane travels further so it takes longer.

## EXAMPLE: A Mass M Sliding Down a Curved Surface (no friction):

Consider a problem like that indicated below


Use the bottom of the inclined plane as the Reference Level for calculating potential energy. The initial height of the mass M above the Reference Level is H so the initial Potential Energy is
$\mathrm{PE}_{o}=\mathrm{MgH}$
The final Potential Energy is given in terms of the height h of the final position of the mass M above the Reference Level

$$
\mathrm{PE}_{f}=\mathrm{Mgh}
$$

Suppose the mass is released from rest so that the initial velocity $\mathrm{V}_{0}=0$ so the initial Kinetic

Energy is $\mathrm{KE}_{o}=\frac{1}{2} M \mathrm{~V}_{o}{ }^{2}=0$. We suppose the final velocity $\mathrm{V}_{f}$ is what we want to calculate but if we knew $\mathrm{V}_{f}$, then the final Kinetic Energy is $\mathrm{KE}_{f}=\frac{1}{2} M \mathrm{~V}_{f}{ }^{2}$. Conservation of mechanical energy $\mathrm{KE}_{o}+\mathrm{PE}_{o}=\mathrm{KE}_{f}+\mathrm{PE}_{f}$ applied to this problem yields
$0+\mathrm{MgH}=\frac{1}{2} M \mathrm{~V}_{f}{ }^{2}+\mathrm{Mgh}$
The mass M cancels out and solving for $\mathrm{V}_{f}$ we get

$$
\mathrm{V}_{f}=\sqrt{2 g(H-h)}
$$

If you think about it a moment, you will realize that the Newton's Law method cannot be used for this problem since the sliding surface is not a plane so the horizontal and vertical components of the gravitational force Mg vary continuously down the sliding surface.

## A Numerical Example:

Suppose $\mathrm{H}=5 \mathrm{~m}$ and $\mathrm{h}=2 \mathrm{~m}$ then the velocity

$$
\begin{aligned}
& H=5 . ; h=2 . ; g=9.8 ; \\
& V f=\sqrt{2 * g *(H-h)} \\
& 7.66812
\end{aligned}
$$

that is, $\mathrm{V}_{f}=7.7 \mathrm{~m} / \mathrm{s}$

## EXAMPLE: A Mass Sliding Down and Inclined Plane with friction:

Consider a mass M sliding down an inclined plane of angle $\theta$ the horizontal and indicated by the diagram below.


Use the bottom of the inclined plane as the Reference Level for calculating potential energy. The initial Potential Energy is $\mathrm{PE}_{o}=\mathrm{Mgh}$ since the mass M is a height h above the reference level initially. Also, you are given the height H of the final position above the Reference Level so the final Potential Energy is $\mathrm{PE}_{f}=\mathrm{Mg} \mathrm{H}$.

Suppose the mass is released from rest so that the initial velocity $\mathrm{V}_{0}=0$ so the initial Kinetic Energy is $\mathrm{KE}_{o}=\frac{1}{2} M \mathrm{~V}_{o}{ }^{2}=0$ and the final Kinetic Energy is $\mathrm{KE}_{f}=\frac{1}{2} M \mathrm{~V}_{f}{ }^{2}$ which can be determined my measuring $\mathrm{V}_{f}$.

## Numerical Example:

Suppose the initial height is $\mathrm{H}=5 \mathrm{~m}$ of the mass $\mathrm{M}=1.5 \mathrm{~kg}$ which is released from rest so $\mathrm{V}_{o}=0$. Also, suppose the final height $\mathrm{h}=2 \mathrm{~m}$ and the final velocity is measured to be $\mathrm{V}_{f}=4 \mathrm{~m} / \mathrm{s}$. The initial Total Mechanical Energy is $\mathrm{TE}_{o}=\mathrm{KE}_{o}+\mathrm{PE}_{o}$ and for this problem $\mathrm{TE}_{o}=73.5$ Joules (see below)

$$
\begin{aligned}
& \mathrm{M}=1.5 ; \mathrm{VO}=0 . ; \mathrm{H}=5 . ; \\
& \mathrm{PEO}=\mathrm{M} * \mathrm{~g} * \mathrm{H} ; \\
& \mathrm{KEO}=\frac{1}{2} * \mathrm{M} * \mathrm{VO}^{2} ; \\
& \mathrm{TEO}=\mathrm{PEO}+\mathrm{KEO} \\
& 73.5
\end{aligned}
$$

The final Total Energy $=\mathrm{KE}_{f}+\mathrm{PE}_{f}=41.4$ Joules (see below) and this is less than the initial $\mathrm{TE}_{o}$.
Since $\mathrm{TE}_{o} \neq \mathrm{TE}_{f}$ Conservation of Mechanical Energy does NOT apply to this problem.

```
M = 1.5; Vf = 4.; h = 2.;
PEf = M * g *h;
```



```
TEf= PEf + KEf
41.4
```

Conservation of Mechanical Energy is replaced by
$\mathrm{KE}_{o}+\mathrm{PE}_{o}=\mathrm{KE}_{f}+\mathrm{PE}_{f}+\mathrm{W}_{\text {friction }}$
which includes $W_{\text {friction }}$ the work done against friction force. The above equation is at least plausible since the final Total Energy is less than the initial Total Energy. The work done against friction would heat up the mass M slightly and this represents Heat Energy. Heat Energy is thought of as the increased speed of the atoms that make up the mass $M$ but since these are random motion, Heat Energy is not thought of as Mechanical Energy like Kinetic Energy and Potential Energy. In principle, if you knew the temperature increase you could determine the work done against friction $\mathrm{W}_{\text {friction }}$. (Later in the semester you will learn how to do this sort of calculation.) Instead here we use the generalized Conservation of Energy equation to calculate the work done against friction

$$
\mathrm{W}_{\text {friction }}=\mathrm{TE}_{o}-\mathrm{TE}_{f}=\mathrm{KE}_{o}+\mathrm{PE}_{o}-\left(\mathrm{KE}_{f}+\mathrm{PE}_{f}\right)
$$

## A Numerical Example:

The various terms on the right hand side of the above equation have been calculated previously and we obtained $\mathrm{TE}_{o}=73.5$ Joules and $\mathrm{TE}_{f}=41.4$ Joules

```
H=5.; g = 9.8; M = 1.5; VO = 0.;
PEO = M * G * H;
KEO = \frac{1}{2}}*M* VO'
TEO = PEO + KEO;
h = 2.; g = 9.8; M = 1.5; VO = 0.;
PEf = M * g * h;
KEf = \frac{1}{2}*M*Vf'2;
TEf = PEf + KEf;
Wf = TEO - TEf
32.1
```

So $W_{f}=32.1$ Joules of work is done against friction.

## Different Forms of Energy:

This part of the physics course involves kinetic energy, potential energy, and heat energy through the work done by friction. This semester deals with gravitational potential energy but next term you will study the potential energy due to the electric force: potential (gravitational,

1. Kinetic Energy (energy of motion)
2. Potential Energy (energy of position)
3. Friction Work or Heat Energy

You may also classify energy according to the means by which it was formed. Examples include Nuclear

Solar
Geothermal
Wind
Chemical

## The difference between "Conservation of Energy" as used in this course and "Energy Conservation" as used by Con Ed and various NYC, NY State or Federal agencies.


#### Abstract

"Energy Conservation" is a term used by Con Ed to indicate energy is saved from being lost and wasted into the environment. This can mean something as simple (but important) as plugging the holes in a home so heat energy is not wasted to the environment. Another example is the use of more efficient light bulbs so more of the electrical energy is converted into light and less is converted into heat which is not useful.


This idea of "energy conservation" is quite a bit different from the Law of Conservation of Energy discussed here. Here energy is converted from one form into another but the total energy (mechanical and non-mechanical) is a constant or is conserved.

## Power:

Power P is defined as the rate of doing work W
$P=\frac{W}{t}$.
UNITS for POWER: Joule/sec = Watt
Quite often you are interested in the rate that work is done or the rate with which energy is utilized.
For example, light bulbs are rated in Watts and you know the higher the Watt rating, the brighter the bulb (A 100 Watt bulb is brighter than a 60 Watt bulb for example).

The Con Edison is an energy company because it is more interested in the total energy (Joules) you utilize each day (of course Con Ed must be able to supply enough energy to meet the peak demand which is power). For example, if you use a light bulb with a power rating of 120 Watts for 3 hours, then the energy supplied by Con Edison is 360 Watt-Hours since
$\mathrm{W}=\mathrm{P} \times \mathrm{t}$
Con Edison charges say 12 cents or $\$ 0.12$ for each Kilo-Watt-Hr= 1,000 Watt-Hr. For example, the cost of having the 120 Watt light bulb on for 3 Hr is about 4 cents.

$$
\begin{aligned}
& \frac{120 . * 3}{1000} * 0.12 \\
& 0.0432
\end{aligned}
$$

For a whole month, the cost of having a 120 Watt bulb 24 Hours/7 Days is about $\$ 10$.


The Watt-Hr is a little awkward as a unit of energy but it is with us for historical reasons.
The more common unit of energy in the S.I. is the Joule.
Example of Joules of Energy and Watts of Power: The amount of energy in which is used in having a 120 Watt $=120$ Joules/sec light bulb on 24 hours/day for a month is $3.7 \times 10^{4}$ Joules.

$$
\begin{aligned}
& \frac{120 .}{1000} *(24 * 30 * 60 * 60) \\
& 37324.8
\end{aligned}
$$

30 Days $\times 24 \frac{\mathrm{Hr}}{\text { Day }} \times 60 \frac{\mathrm{Min.}}{\mathrm{Hr} .} \times 60 \frac{\mathrm{Sec} .}{\mathrm{Min} .}=2.6 \times 10^{6} \mathrm{sec}$
is the number of seconds in a month.

30 * 24 * 60 * 60
2592000
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