

# 10. The Series Resistor and Inductor Circuit

## *Introduction*

The last laboratory involved a resistor, R and capacitor, C in series with a battery switch on or off. It was simpler, as a practical matter, to replace the battery and switch with a signal generator producing a square wave. The current and voltage across the resistor and capacitor in the circuit were calculated and measured.

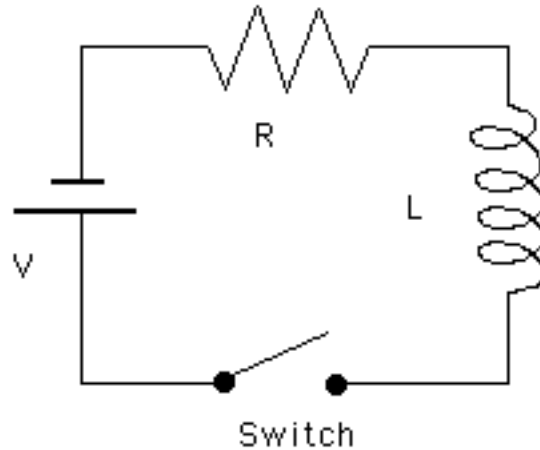
This lab involves a resistor, R and inductor, L in series with a battery and switch and the current is calculated as a function of time. Also, the voltage drops across the resistor and inductor are calculated. Again it is easier to study an experimental circuit with the battery and switch replaced by a signal generator producing a square wave.

## *Kirchoff's Loop Rule for a RL Circuit*

The voltage,  $V_L$  across an inductor, L is given by

$$V_L = L \cdot \frac{d}{dt} i [t] \quad (1)$$

where  $i[t]$  is the current which depends upon time, t. (Sometimes there is a minus sign placed in front of the right hand side and this is called "Lenz's Law" but we shall not need worry about this as long as we treat equation like a voltage **drop** across a resistor.) Equation (1) results from Faraday's Law of electricity and magnetism. Consider the resistor and inductor circuit below:



### *Solution of the Loop Equation*

NOTE: Reading this for the first time, you might want to skip this section and go right to the solution equation (11). You can verify that equation (11) is the solution by substituting equation (11) into equation (2).

The Kirchoff Loop Rule applied to the circuit above after the switch is closed is

$$i[t] R + L \frac{d}{dt} i[t] = V \quad (2)$$

After a long time the current stops changing and so  $di[t]/dt=0$  and equation (2) reduces to  $i[t \rightarrow \infty]R=V$  which can be solved for the current after a long time  $i[t \rightarrow \infty]$ . The current initially right after the switch is closed is zero  $i[0]=0$ .

Equation (2) is easily solved by first dropping the constant  $V$  term on the right hand side so that you get

$$i[t] R + L \frac{d}{dt} i[t] = 0 \quad (3)$$

Equation (3) can be rearranged

$$\frac{di}{i} = -\frac{R}{L} dt \quad (4)$$

which is easily integrated

$$\int \frac{di}{i} = -\frac{R}{L} \int dt \quad (5)$$

to obtain

$$\text{Log}[i] = -\frac{R}{L}t + C \quad (6)$$

where  $C$  is an integration constant. Using a property of Logarithms equation (6) is equivalent to

$$i[t] = \bar{C} \text{Exp}\left[-\frac{R}{L}t\right] \quad (7)$$

where  $\bar{C}$  is a constant related to  $C$  so we have the solution to equation (3). But what we want is the solution to equation (2) which is a little different. We make a guess that the solution to equation (2) is somewhat like equation (7)

$$i[t] = f[t] \text{Exp}\left[-\frac{R}{L}t\right] \quad (8)$$

where the coefficient  $f[t]$  is not constant. Note that since  $i[0]=0$  it follows from equation (8) with  $t=0$  that  $f[0]=0$ .  $f[t]$  is determined by substitution of equation (8) into equation (2). After doing this one gets

$$\frac{d}{dt} f[t] = \frac{V}{L} \text{Exp}\left[\frac{R}{L}t\right] \quad (9)$$

Equation (9) is easily integrated to yield

$$f[t] = f[0] + \frac{V}{R} \left( \text{Exp}\left[\frac{R}{L}t\right] - 1 \right) \quad (10)$$

and remember  $f[0]=0$ . Finally using equation (10) in equation (8) yields

$$i[t] = \frac{V}{R} \left( 1 - \text{Exp}\left[-\frac{R}{L}t\right] \right) \quad (11)$$

Note that from equation (11)  $i[0]=0$  and  $i[\infty]=V/R$  as we expect. An inductive time constant  $\tau=L/R$  is sometimes introduced and equation (11) becomes

$$i[t] = \frac{V}{R} \left( 1 - \text{Exp}\left[-\frac{t}{\tau}\right] \right) \quad (12)$$

**Note:**  $\tau=L/R$  is the time constant for the LR circuit. This time constant  $\tau$  is different from the time constant for the RC circuit where  $\tau=RC$ . The same symbol  $\tau$  is used in both the RL and RC circuits but obviously the meaning is different. The voltage across the resistor is just  $i[t]*R$  that is

$$V[t] = V \left( 1 - \text{Exp}\left[-\frac{t}{\tau}\right] \right) \quad (13)$$

### *Graph of the Solution Equation (11)*

Suppose  $R=8k\Omega=8000\Omega$  and  $L=6\text{mH}=0.006\text{H}$  and the battery voltage is  $V=10$  volts. We use equation (11) to graph the current  $i[t]$  versus time  $t$ .

```

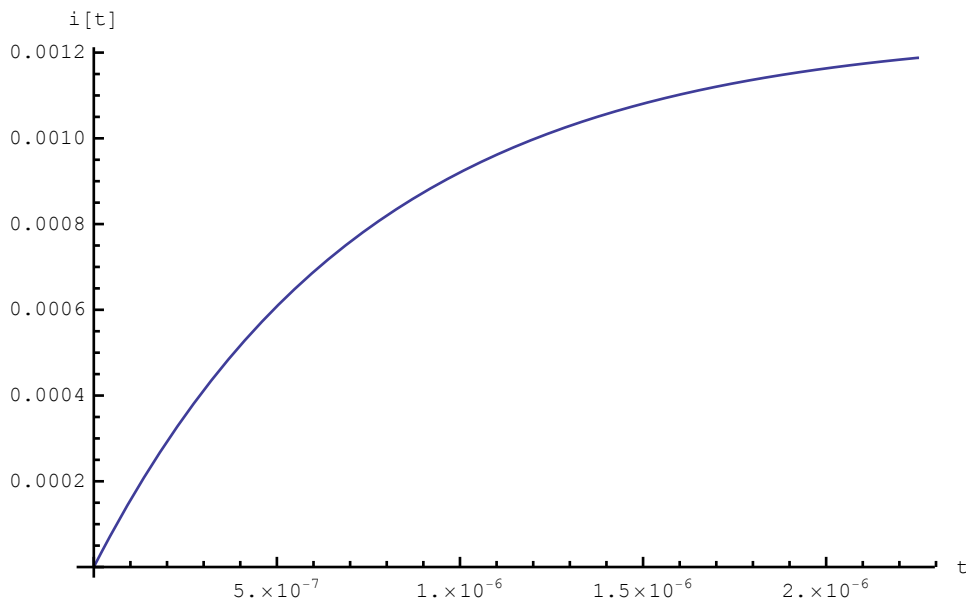
R = 8000.;
L = 0.006;
τ = L / R; Print["τ = ", τ]
V = 10.;
i[t_] := V / R * (1 - Exp[-t / τ])

τ = 7.5 × 10-7

```

so the time constant  $\tau=0.75 \mu\text{sec}$  for this LR circuit.

```
Plot[i[t], {t, 0, 3 * τ}, AxesLabel -> {"t", "i[t]"}]
```



The maximum value of the current is

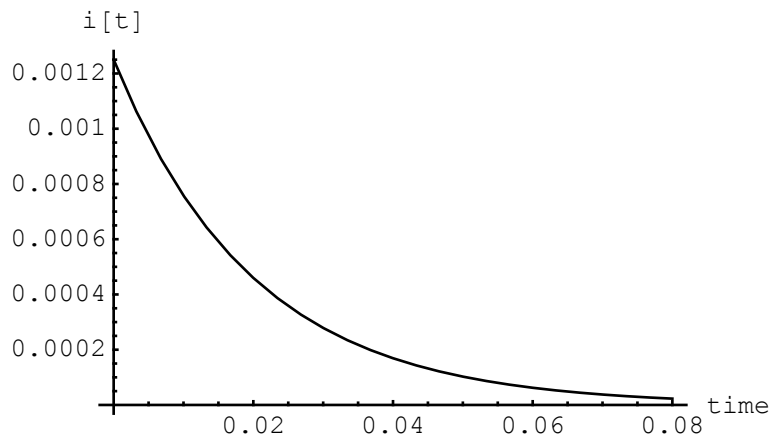
```

V / R
0.00125

```

and after one time constant  $\tau=7.7 \times 10^{-6}$  sec the current is about 66% of that value. The voltage across the resistor is just  $i[t]*R$  so the voltage across the resistor has the same shape as the above  $i[t]$  versus time graph but the vertical scale is different. You can measure the voltage across the resistor with your oscilloscope.

Aside: You should compare the above graph to the current (and voltage) across a resistor in an **RC circuit**



This graph is quite a bit different than for the current in the LR circuit. For the RC circuit, the current starts large at  $t=0$  and the current goes to zero for long times while the opposite behavior is true for the current in the LR circuit since then the current starts out small and rises to its maximum value.

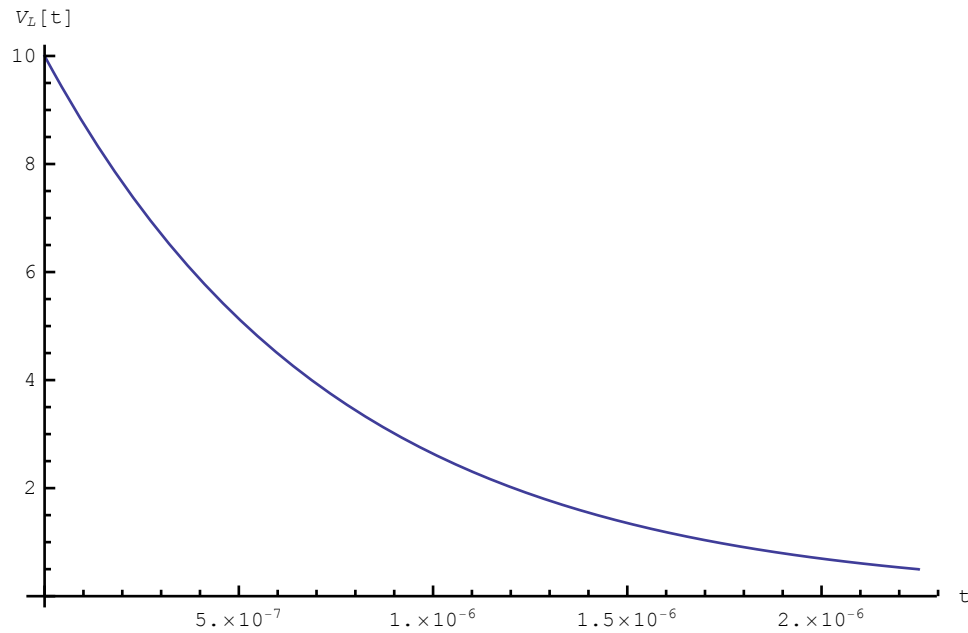
Return to the RL circuit. The voltage across the inductor is obtained by taking the time derivative of equation (12) and using equation (10) to get

$$V_L[t] = L \frac{V}{R\tau} \text{Exp}\left[-\frac{t}{\tau}\right] = V \text{Exp}\left[-\frac{t}{\tau}\right] \quad (14)$$

A graph of the voltage across the inductor equation (13) appears

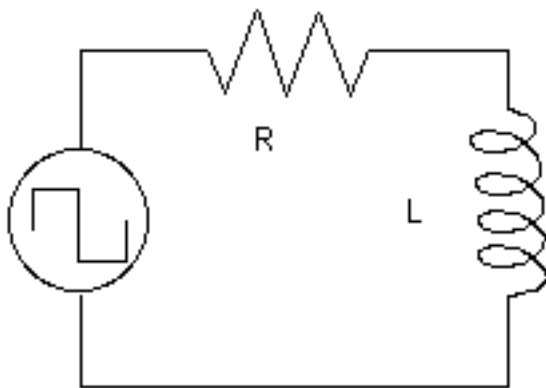
$$V_L[t_] := V * \text{Exp}\left[-\frac{t}{\tau}\right]$$

```
Plot[VL[t], {t, 0, 3 * τ}, AxesLabel → {"t", "VL[t]"}]
```



### *Replace the Battery and Switch by a Signal Generator having a Square Wave*

The series RL circuit diagram now appears



Suppose the  $L$  and  $R$  are such that the time constant  $\tau=0.75 \mu\text{sec}$  as before. As a start, take the period  $T$  of the signal generator to be the same as  $\tau$  so the frequency  $f$  of the signal generator is

```

τ = 0.75 × 10-6;
T = τ;
f = 1 / T

```

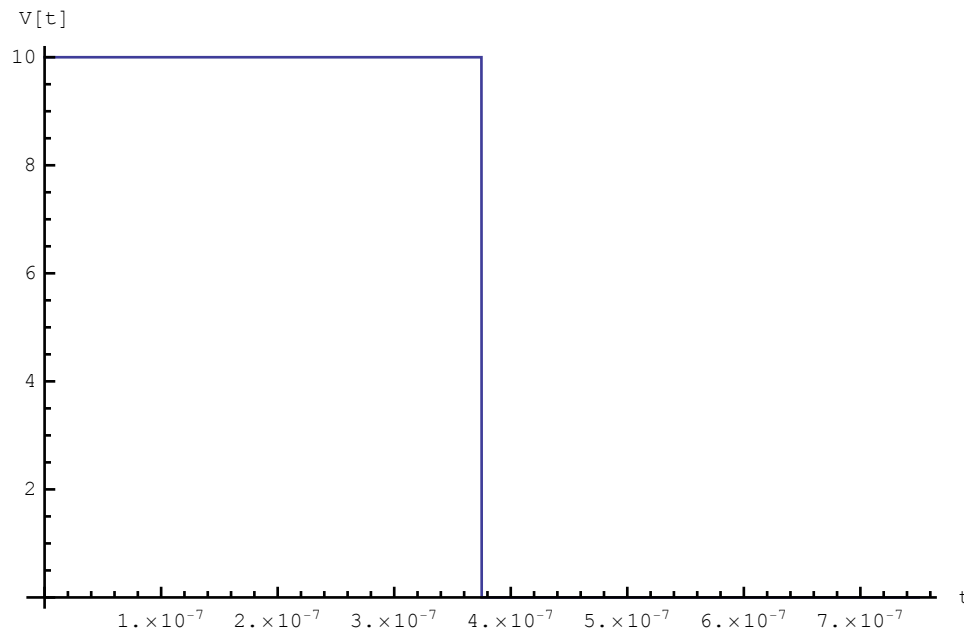
```
1.33333 × 106
```

so the signal generator is set to 1.3 megaHz. Suppose the voltage amplitude of the signal generator is 10 volts (the same as the battery voltage) The square wave of the signal generator is graphed using

```

V0 = 10.;
SqWave[t_] := If[t <  $\frac{T}{2}$ , V0, 0]
Plot[SqWave[t], {t, 0, T}, AxesLabel → {"t", "V[t]"}]

```



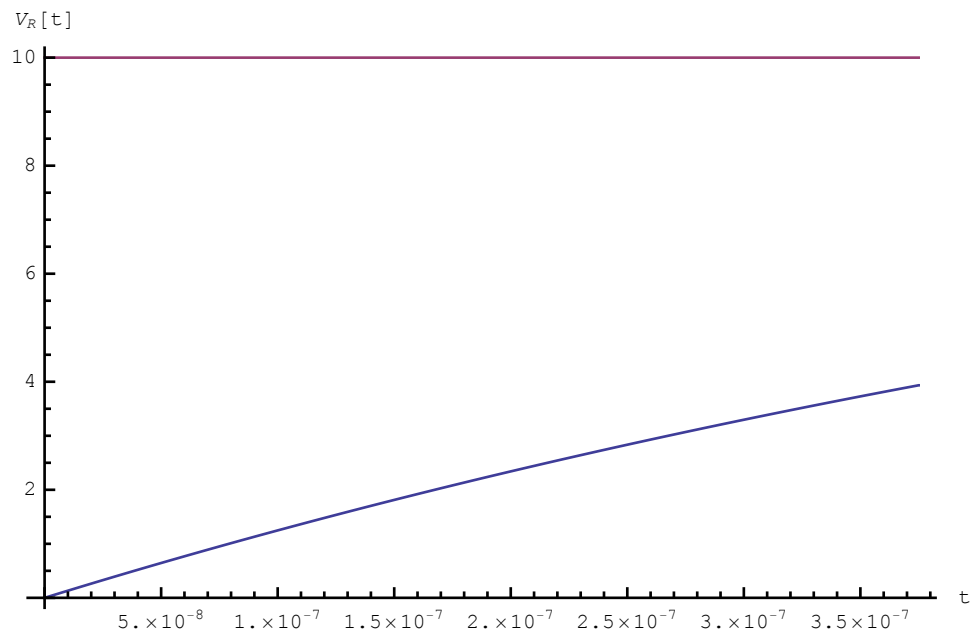
Effectively having the signal generator in the circuit is the same as having the battery in the circuit for time  $0 < t < 0.4 \mu\text{sec}$ .

## *The Voltage Across the Resistor*

We may graph the voltage across the resistor equation (12) together with the signal generator voltage and obtain

$$V[t_] = V_0 \left( 1 - \text{Exp}\left[-\frac{t}{\tau}\right] \right);$$

```
Plot[{V[t], SqWave[t]}, {t, 0,  $\frac{T}{2}$ }, AxesLabel -> {"t", "VR[t]"}]
```

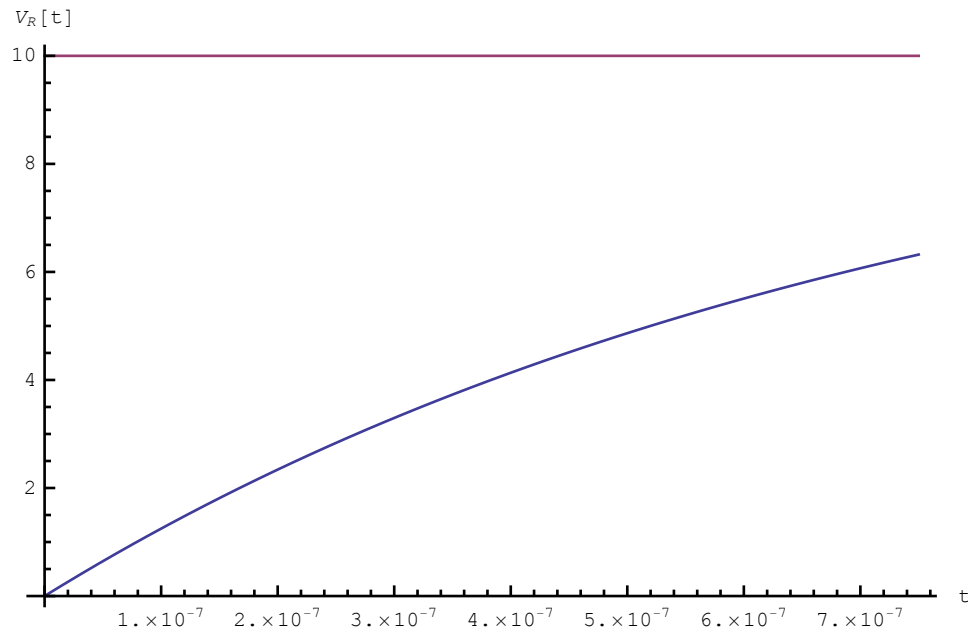


If the period  $T$  of the signal generator is longer, for example  $T=2*\tau$ , then the current in the circuit has more time to increase



$$T = 2 \tau;$$

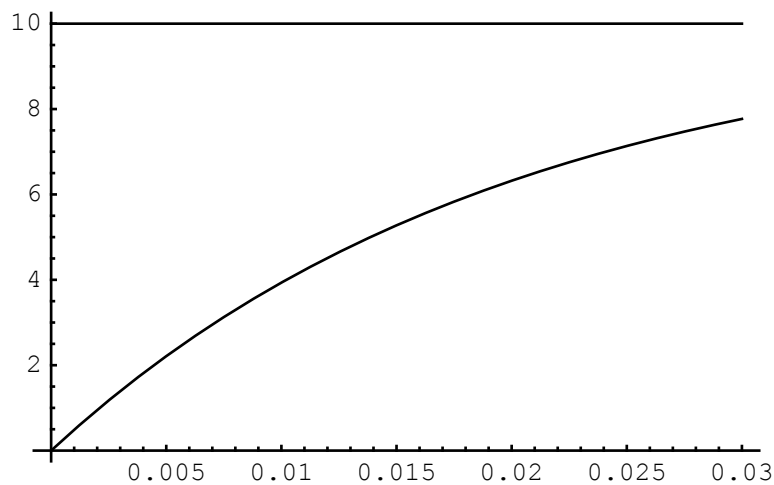
```
Plot[{V[t], SqWave[t]}, {t, 0,  $\frac{T}{2}$ }, AxesLabel -> {"t", "VR[t]"}]
```



Further if the period  $T$  of the signal generator is longer still, for example  $T=3*\tau$ , then the current in the circuit has even more time to increase

$$T = 3 \tau;$$

```
Plot[{V[t], SqWave[t]}, {t, 0,  $\frac{T}{2}$ }, AxesLabel -> {"t", "VR[t]"}]
```



So after a time equal to three time constants  $\tau$ , the current is about 8 Volts and is approaching the maximum value of 10 Volts. After a sufficiently long time, the current in the circuit is almost constant so the

time rate of change of the current in the LR circuit becomes zero after a long time. As a result the voltage across the inductor approaches zero after a long time and all the signal generator voltage is the same as the voltage across the resistor.

This increasing behavior of the current with time in the LR circuit should be **contrasted** with the decreasing current in the RC circuit with time. For the RC circuit, initially there is no charge on the capacitor and since the voltage across the capacitor is  $Q/C$ , the voltage initially across the capacitor is zero. After a long time, the capacitor charges up to the maximum charge and the voltage across the capacitor increases to its maximum value.

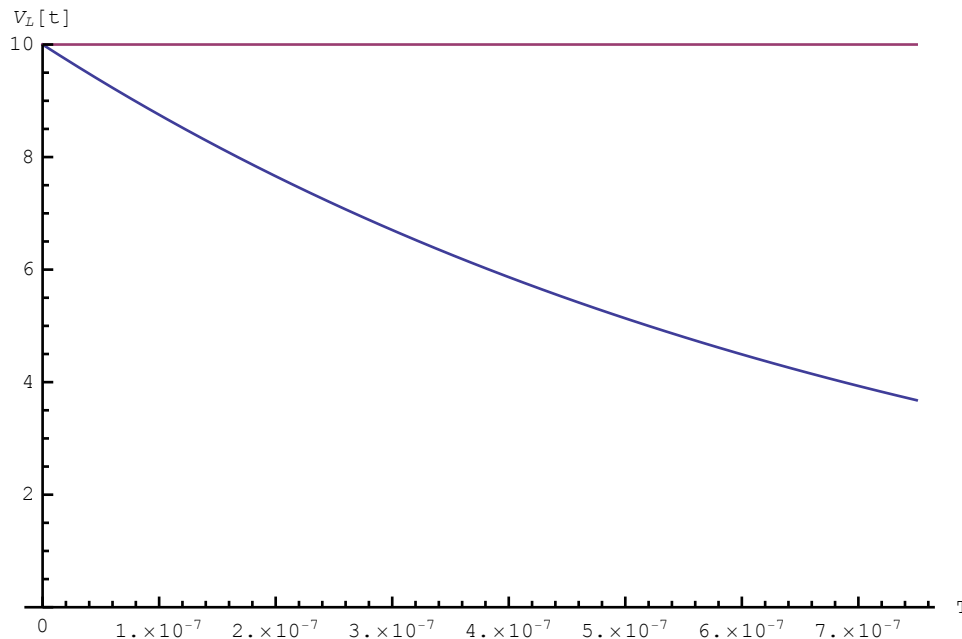
## *The Voltage Across the Inductor*

We may graph the voltage across the inductor  $V_L$  equation (14) together with the signal generator voltage and obtain

```

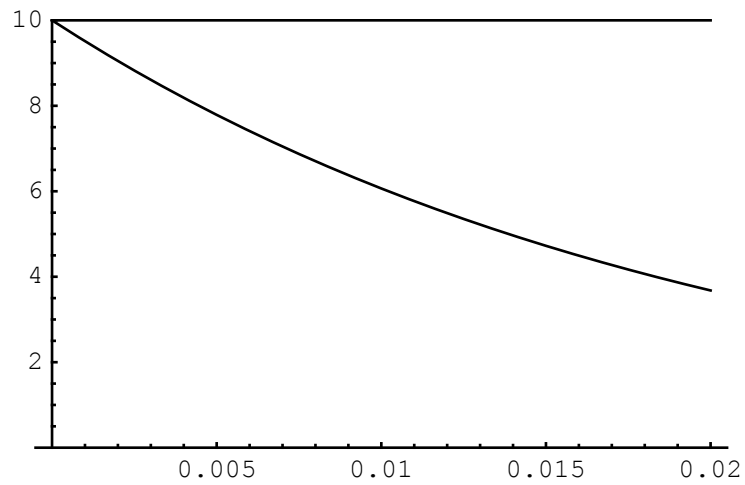
VL[t_] := V0 * Exp[- $\frac{t}{\tau}$ ]
Plot[{VL[t], SqWave[t]}, {t, 0,  $\frac{T}{2}$ },
PlotRange -> {0, 10}, AxesLabel -> {"T", "VL[t]"}]

```



So the voltage across the inductor is decreasing since  $di/dt$  is decreasing. If the period  $T$  of the signal generator is longer, for example  $T=2*\tau$ , then the current gets smaller still and the voltage across the inductor is reduced further

```
T = 2 τ;  
Plot[{V_L[t], SqWave[t]}, {t, 0,  $\frac{T}{2}$ },  
PlotRange -> {0, 10}, AxesLabel -> {"T", "V_L[t]"}]
```

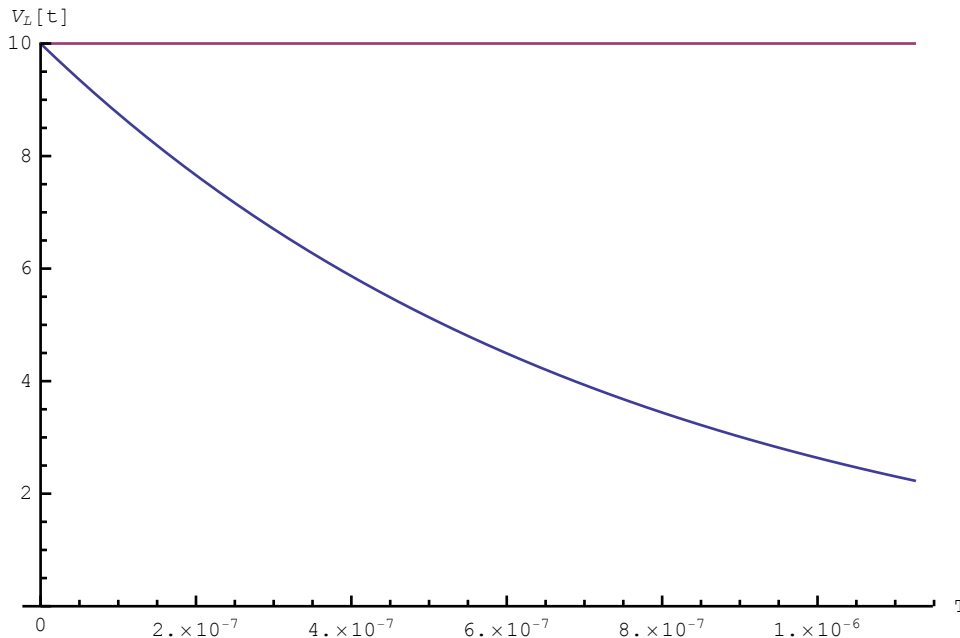


If the period  $T$  of the signal generator is even longer, for example  $T=3\tau$ , then the current gets smaller still and the voltage across the inductor is reduced further

```

T = 3 τ;
Plot[{VL[t], SqWave[t]}, {t, 0,  $\frac{T}{2}$ },
  PlotRange → {0, 10}, AxesLabel → {"T", "VL[t]"}]

```



## Laboratory Exercises

**PART A:** Place a signal generator in series with a resistor and inductor. Pretty much any output level (the output voltage) of the signal generator will do OK but after you get the oscilloscope working properly make a note of the maximum voltage in your lab notebook. Choose a **square wave** and make the frequency  $f$  of the signal generator such that  $f = \frac{1}{T}$  with  $T = \tau = L/R$  at first. With channel 1 of the oscilloscope, measure the voltage across the signal generator and with channel 2 measure the **voltage across the inductor**. Compare with the graphs of the first example above. Make the frequency  $f$  of the signal generator smaller ( $T$  larger) so the current in the circuit has more time to increase toward its maximum (the voltage across the inductor decreases). The voltage across the inductor decreases with time. Keep decreasing  $f$ . Sketch the oscilloscope figures you get and indicate the values of the voltage on the vertical scale and the time on the horizontal scales.

Example: Suppose  $L = 6$  mH and  $R = 8.0$  k $\Omega$  then the time constant  $\tau = L/R = 0.76$   $\mu$ sec. as indicated below:

$$\begin{aligned} R &= 8000. ; \\ L &= 0.006 ; \\ \tau &= L / R \end{aligned}$$

$$7.5 \times 10^{-7}$$

which is 0.75  $\mu\text{sec}$  and your oscilloscope should be set for something like 0.2  $\mu\text{sec}$  per division (but this might not be accessible to your oscilloscope). Your signal generator  $f$  is

$$\begin{aligned} T &= \tau ; \\ f &= 1 / T \end{aligned}$$

$$1.33333 \times 10^6$$

and the signal generator frequency should be 1.3 MHz.

NOTE: The value of  $R$  and  $L$  you use need not be the values given above. It probably would be a good idea to take the largest inductance you have say 40 mH since then  $\tau$  larger

$$\begin{aligned} R &= 8000. ; \\ L &= 0.04 ; \\ \tau &= L / R \end{aligned}$$

$$5. \times 10^{-6}$$

and your oscilloscope should be set for something like 2  $\mu\text{sec}$  per division. Your signal generator  $f$  is

$$\begin{aligned} T &= \tau ; \\ f &= 1 / T \end{aligned}$$

$$200000.$$

or 200 kHz which is easily accessible by your signal generator.

It is not a good idea to take the resistor too small since then the current is too large and this might overload your signal generator and cause the output signal to be distorted.

Use your digital ohmmeter to measure the value of the resistor and make sure it is the same as given by the color code. Use your digital inductance meter to measured the value of the inductance and it should agree with the value for  $L$  printed on the inductor.

**PART B:** With channel 1 of the oscilloscope, measure the voltage across the signal generator and with channel 2 measure the voltage across the resistor. Compare with the graphs of the second example above. Make the frequency  $f$  of the signal generator smaller ( $T$  larger) so the capacitor has more time to

charge. Keep decreasing  $f$  the frequency of the signal generator. Sketch the oscilloscope figures you get.

**PART C:** Call the inductor used above  $L_2$ . Take a second inductor and call it  $L_2$ . Combine the two inductors in SERIES without the signal generator and oscilloscope attached. The effective capacitance is given by

$$L_{\text{eff}} = L_1 + L_2$$

Compute the numerical value of the effective inductance and check it with the digital inductance meter. Use the SERIES combination of  $L_1$  and  $L_2$  together with the signal generator and oscilloscope and repeat the measurements of **PART A** above.

**PART D:** Combine the two capacitors in PARALLEL without the signal generator and oscilloscope attached. The effective capacitance is given by

$$\frac{1}{L_{\text{eff}}} = \frac{1}{L_1} + \frac{1}{L_2}$$

Compute the numerical value of the effective inductance and check it with the digital inductance meter. Use the PARALLEL combination of  $L_1$  and  $L_2$  together with the signal generator and oscilloscope and repeat the measurements of **PART A** above.