**Answer all Questions**

# FIRST MID-TERM

Thursday September 28, 2017

**STUDENT'S NAME:**

1. (i) Find $\nabla\phi$ if

(a) $\phi = \ln|\mathbf{r}|$.

(b) $\phi = \frac{1}{|\mathbf{r}|}$.

Here, $|\mathbf{r}| = \sqrt{x^2 + y^2 + z^2}$.

(ii) Express $\delta(ax)$ in terms of $\delta(x)$ where $a$ is a constant and can be either positive $(a > 0)$ or negative $(a < 0)$ . Justify your answer.

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2. Given

$$\mathbf{F} = \frac{k_1}{\rho}\hat{\rho} + k_2 z \ \hat{z} \ , \tag{1}$$

in cylindrical coordinates with $k_1$ and $k_2$ constants, evaluate the scalar surface integral

$$\oint \mathbf{F} \cdot d\mathbf{a} \tag{2}$$

over the surface of a closed cylinder around the $z$ axis specified by $z = \pm 3$ and radius $R = 2$.

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3. (a) Under what conditions does $\mathbf{A} \times (\mathbf{B} \times \mathbf{C}) = (\mathbf{A} \times \mathbf{B}) \times \mathbf{C}$?

(b) Does $\mathbf{A} \cdot \mathbf{B} = \mathbf{A} \cdot \mathbf{C}$ imply $\mathbf{B} = \mathbf{C}$? Explain.

Using the Levi-Civita epsilon notation ($\{\epsilon_{ijk}\}$) and the Einstein summation convention, prove that

(c) $(\mathbf{A} \times \mathbf{B}) \cdot (\mathbf{C} \times \mathbf{D}) = (\mathbf{A} \cdot \mathbf{C})(\mathbf{B} \cdot \mathbf{D}) - (\mathbf{A} \cdot \mathbf{D})(\mathbf{B} \cdot \mathbf{C})$.

(d) $\nabla \times (\nabla \times \mathbf{A}) = \nabla(\nabla \cdot \mathbf{A}) - \nabla^2 \mathbf{A}$, for arbitrary vector $\mathbf{A}$.

In this notation, $\nabla^2 = \nabla \cdot \nabla$.

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4. A static charge distribution produces a radial electric field

$$\mathbf{E}(r) = A\,\frac{e^{-kr}}{r^2}\ , \tag{3}$$

where $A$ and $k$ are constant.

For this, the charge distribution is

$$\rho(r) = 4\pi\epsilon_0 A\delta(r) - \frac{\epsilon_0 Ak}{r^2}e^{-kr}\ .$$

Using this result,

(a) Determine what is the total charge $Q$ in all space?

(b) This total charge can also be obtained from Gauss's flux theorem

$$Q = \oint_S \epsilon_0 \mathbf{E}\cdot d\mathbf{a}\ .$$

Obtain $Q$ using this method in (b). Here, $S$ is a sphere of radius $R \to \infty$.

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5. Determine the electric field $\mathbf{E}(\mathbf{r})$ as a function of the radial distance $r$ caused by a spherical cloud of electrons with a volume charge density $\rho(r) = -\rho_0$ (a constant) for $0 \leq r \leq R$ (both $\rho_0$ and $R$ are positive) and $\rho(r) = 0$ for $r > R$.

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