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EXCITATION OF ISOBARIC ANALOGUE STATES IN THE ^{27}Al(d, n)^{28}Si REACTION

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Abstract: A high-resolution study of the ^{27}Al(d, n) reaction has revealed five $T = 0$ and four $T = 1$ states formed prominently through $s_{\frac{1}{2}}$ transfer. The absolute spectroscopic factors satisfy the isospin sum rules of French and Macfarlane. The centroids of the $T = 0$ and $T = 1$ components differ by 3.5±0.3 MeV. For comparative purposes, some spectroscopic factors have also been extracted from previously published low-resolution data on the Al(d, n) and Al(d, p) reactions.

E NUCLEAR REACTION ^{27}Al(d, n), $E = 2.980$ MeV; measured $\sigma(E_n, \theta)$. ^{28}Si deduced levels, J, π, T, spectroscopic factors for $s_{\frac{1}{2}}$ transfer.

1. Introduction

The combination of high resolution for low-energy neutrons and low background in time-of-flight neutron spectroscopy is ideal for the study of isobaric analogue states via the (d, n) reaction [1]). The isobaric analogue states thus populated are identified by comparison with the positions, l-values and spectroscopic factors of levels excited in the (d, p) reaction to the analogue nucleus. In particular, the absolute spectroscopic factors should satisfy [2]) the condition

$$S_{(d,n)} = S_{(d,p)}$$

for transitions to analogue states.

In this paper we report a study of the transitions in the ^{27}Al(d, n) reaction having $l_p = 0$. It forms part of a study of the excitation of isobaric analogue states in the 1d-2s shell by the (d, n) reaction.

Wilkinson's [3]) estimate of the position of the lowest state with $T = 1$ in ^{28}Si, 9.384±0.017 MeV, was supported by evidence from the β-decay studies of Glass and Richardson [4]), who estimated the position of this state to be 9.16±0.17 MeV. Calvert *et al.* [5]) used the ^{27}Al(d, n)^{28}Si reaction to give a value of 9.37±0.15 MeV. More recent work by Hinds and Middleton [6]), using the (^{3}He, d) reaction revealed levels at 9.31±0.01 and 9.38±0.01 MeV, which probably correspond to the 0.00 and 0.03 MeV doublet in ^{28}Al. Thus, since the Q-value for the reaction ^{27}Al(d, n)^{28}Si g.s. is 9.353 MeV, reactions leading to isobaric analogues of the low-lying states of ^{28}Al have zero or negative Q-values.

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2. Experimental procedure

The ^{27}Al(d, n)^{28}Si reaction has been studied using the time-of-flight facilities of the Harwell 3 MeV pulsed accelerator IBIS. At the maximum deuteron bombarding energy, the neutrons populating the $T = 1$ states in ^{28}Si have energies of less than 3.0 MeV. In order to study these states for a range of excitation up to 2–3 MeV above the ground state analogue, the bias on the neutron detector [7]) was set such that neutrons down to 150 keV energy could be detected.

As only a 22° particle analysing magnet was available, an independent determination was made of the bombarding energy. It involved a precise measurement of the energy of the group of neutrons from the ^{16}O(d, n)^{17}F reaction leading to the first excited state. The spectrum from this reaction was observed with the neutron detector a short distance d_1 from the target. The detector was then moved away from the target to a position d_2 where the time-of-flight of the neutron had increased by nearly an exact machine period (1000 ns). Since timing was relative to the machine pulses the apparent positions of the peak was only a few channels (Δn) away from its original position. The time-of-flight from d_1 to d_2 was then

$$T(\text{ns}) = 1000.0 + \Delta n \times (\text{time per channel}).$$

The period of the machine can be determined to a very high accuracy by a frequency measurement, and d_2 can be adjusted to make Δn small, therefore the error in the "time per channel" contributes very little to the overall uncertainty. Thus the neutron energy can be precisely determined from the measured distance d_2–d_1 and the calculated time-of-flight.

Allowing for errors in our knowledge of the Q-value of the ^{16}O(d, n)^{17}F* reaction, the bombarding energy was found to be 2.980±0.005 MeV. At the flight path used in the experiment, the time-of-flight spectra were calibrated using lines of known Q-value from the reactions ^{9}Be(d, n), ^{12}C(d, n), ^{16}O(d, n).

Fig. 1 shows a spectrum of neutrons at 0° from the ^{27}Al(d, n)^{28}Si reaction with a target thickness of 150 μg/cm^2. A flight path of 6 m was maintained throughout this work. The advantage of using neutron time-of-flight spectroscopy to provide an expanded energy scale in the region of highly excited states is clearly shown in fig. 1. Angular distributions were measured at 10° intervals in the forward direction and at larger intervals elsewhere. An angular distribution which is typical of a low-Q transition with $l_p = 0$ is shown in fig. 2. The energies of the levels observed in the (d, n) reactions are summarized in table 1, column 1. The upper states listed are thought to be isobaric analogues of the states in ^{28}Al whose excitation energies determined in the (d, p) reaction are listed in column 9. The excitation energies of the postulated analogue states relative to the ground-state analogue are given in column 3.

The absolute differential cross section (column 4) was estimated from a measurement of the yield from a thick target ($\approx$ 3.5 mg/cm^2) in two ways:

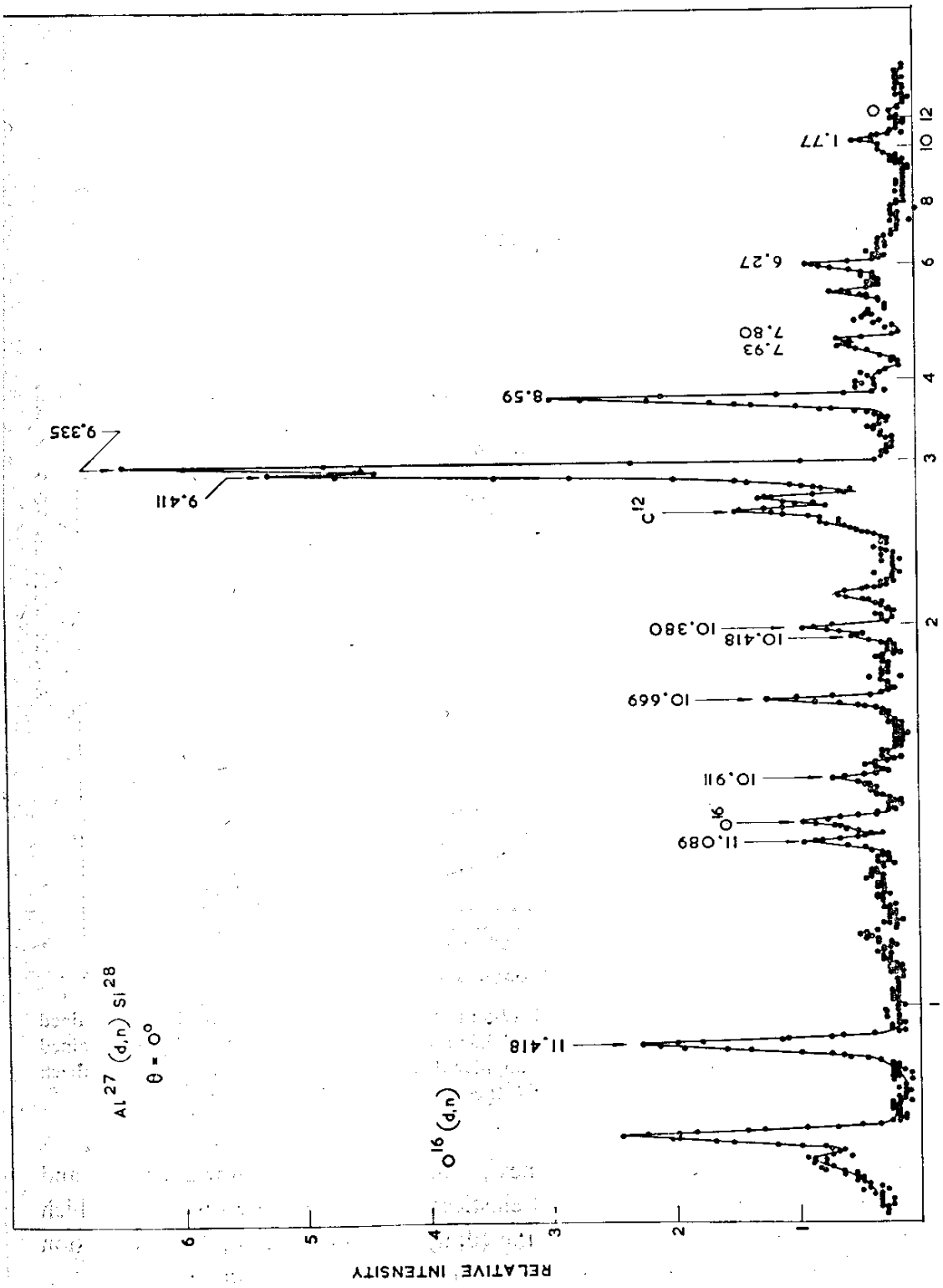

Fig. 1. Neutron spectrum at 0° from the ^{27}Al(d, n)^{28}Si reaction (6 m flight path).

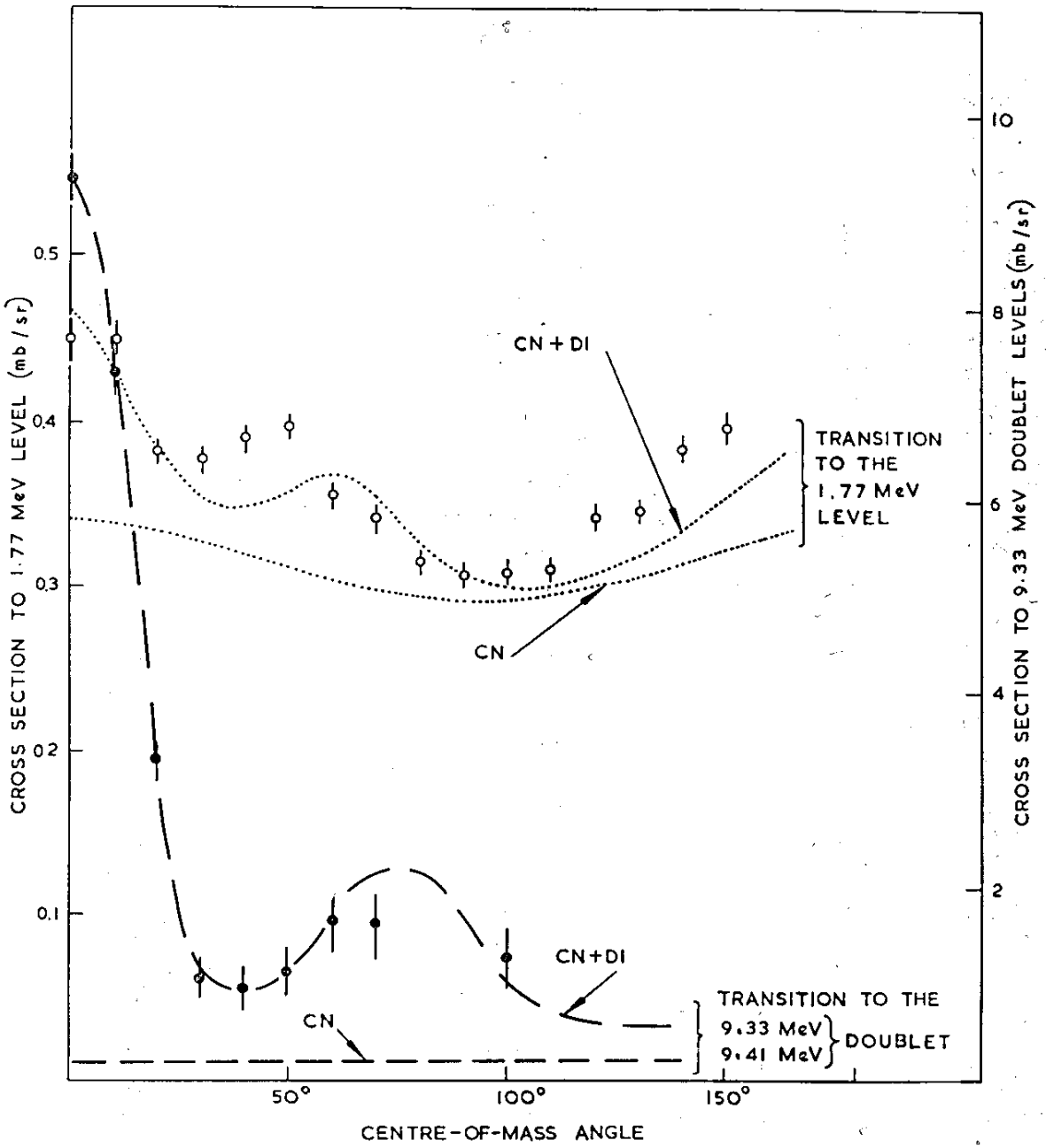

Fig. 2. Angular distribution of neutron groups leading to the 1.77 MeV state (open circles) obtained with a target ≈ 500 keV thick and to the combined 9.33 and 9.41 MeV states (filled circles) obtained with a target ≈ 50 keV thick. The curves show angular distribution predicted by Hauser-Feshbach and DWBA theories.

(i) from beam intensities, target thickness, solid angle and detector efficiency, and

(ii) from a comparison of the yield of elastically scattered deuterons at 30° which could be measured at the same time as the (d, n) spectrum; the (d, d) cross section was estimated by an optical-model code. Both methods agreed within 30 %.

For states with $E_x > 9.4$ MeV, the thin target yield was used with appropriate normalization.

TABLE 1

Summary of measurements for $l = 0$ angular momentum transfer

^{28}Si* (MeV)	T	$E_x - 9.335$ (MeV)	$d\sigma$ [a] (mb/sr)	$d\sigma_{CN}$ [b] (mb/sr)	$(2J+1)S_pC^2$ Potential A	$(2J+1)S_pC^2$ Potential B	$(2J+1)S_pC^2$ Potential C [c]	^{28}Al* (MeV)	$(2J+1)S_nC^2$ Potential C [d]
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
1.77 ±0.05	0		0.45	0.34	0.85±0.2	1.0±0.2	0.65		
6.27 ±0.05	0		0.74	0.26	0.75±0.1	0.9±0.1	1.04		
7.80 ±0.05	0		1.16	0.40	0.65±0.1	0.8±0.1	1.53		
7.93 ±0.05	0								
8.59 ±0.05	0		3.14	0.18	1.7 ±0.2	2.1±0.2			
9.335±0.010	1	0.0	9.35	0.32	3.9 ±0.4	4.7±0.4	3.38	0.0	5.1±0.7
9.411±0.010	1	0.076						0.031	
10.380±0.010	1	1.045	0.72 [e]	0.12	0.2 ±0.1	0.3±0.1		1.017	0.36
10.418±0.010	?	1.083	0.15 [e]	0.12	≈0.04	≈0.06			
10.669±0.010	?	1.334	0.6 [e]	0.11				1.375 ($l = 2$)	
11.418±0.010	1	2.083	1.07	0.08	0.6 ±0.1	0.7±0.1		2.147	1.1±0.2
higher states	1				≈0.5	≈0.6			≈0.7
					$G_p(T<) = 0.33\pm0.04$	$G_p(T<) = 0.40\pm0.05$	$G_p(T<) = 0.27\pm0.03$		$G_n = 1.2\pm0.2$
					$G_p(T>) = 0.44\pm0.04$	$G_p(T>) = 0.53\pm0.06$			

[a]) Absolute differential cross section at 0°; errors ± 15 %.

[b]) Compound nucleus formation cross section estimated by means of the Hauser-Feshbach theory and the thick target angular distributions; error ±30 %.

[c]) Ref. [5]), $E_d = 9$ MeV.

[d]) Ref. [14]), $E_d = 15$ MeV and also ref. [15]).

[e]) $l = 2$ admixture also present.

3. Discussion

3.1. OPTICAL-MODEL PARAMETERS

It is now generally believed [8]) that the depth of the real part of the optical potential for deuteron scattering is $\approx$ 100 MeV. The potential seen by the incoming deuteron wave has a major influence on the DWBA cross sections. The (Al+d) potential is however not known for $E_d \approx 3$ MeV. We use published Al(d, d) results [9]) to search for Saxon-Woods type potentials at $E_d = 3.32$, 4.07 and 11.8 MeV. The total reaction cross sections can be estimated from previous experiments on the Al(d, d'), (d, p) and (d, α) reactions [10]) including a compound elastic cross section of 100 mb (see table 2). The optical-model transmission coefficients tend to give too high σ_R; the least-squares fitting procedure in the optical-model code was therefore made to include σ_R. The "best" potentials are summarized in table 2.

TABLE 2
Optical-model potentials for deuteron scattering on Al

Type	E_d (MeV)	U (MeV)	r_u (MeV)	a_u (fm)	$-W$ (MeV)	r_w (fm)	a_w (fm)	σ_R theory	σ_R expt
A	3.32	120	1.04	0.77	24.9	1.50	0.45	460	350±100
B	4.07	120	1.11	0.77	13.9	1.61	0.68	830	500±300
C	11.8	120	1.17	0.73	21.1	1.63	0.65	1300	

Potential A is taken on the Coulomb barrier and the others above it. We treat the difference between the predictions of calculations using potentials A and B as an experimental uncertainty in the reduction of our results. The various reaction cross sections implied by both potentials may be too high however since the σ_R is poorly given.

3.2 THE ROLE OF CN FORMATION

The compound nucleus formation cross section was calculated by means of the Hauser-Feshbach formalism [11]) taking explicitly into account the various open channels in the (d, n), (d, p), (d, α) and (d, d') reactions. Although 100 channels were so considered, the HF estimate was still likely to be an overestimate since many channels are missed at high excitation and also, as suggested by Kuehner *et al.* [12]), due to the depletion of the incoming deuteron wave by the stripping process.

For the (d, n) reaction, the predicted magnitude of this cross section was small compared with the observed except in the case of the group populating the 1.77 MeV state.

If a large number of compound nuclear states are excited so that a proper average is obtained, one may simply write

$$\sigma_{obs} = \sigma_{CN} + \sigma_{DI},$$

as one expects the interference of the two reaction processes to cancel. In our case, the averaging was achieved by performing a separate measurement using a target

of thickness $\approx$ 500 keV which still permits the low-lying states to be well resolved. The angular distribution of the neutron group leading to the 1.77 MeV state obtained in the measurements is shown in fig. 2. The angular distribution of the CN process calculated as described above was normalized to the observed cross section at around 90°, and the direct interaction component was obtained by subtraction. The same normalization was applied to the calculated compound nucleus cross sections for all the other levels, and these are the values shown in table 1, column 5.

3.3 SPECTROSCOPIC FACTORS EXTRACTED BY DWBA

With $E_D = 3$ MeV, the deuteron is barely above the Coulomb barrier, and the DWBA calculations may not be very reliable. The DWBA code is used to cover a wide range of Q-values ($-2 < Q < +8$ MeV). The function $\delta\sigma_{DWBA}(Q)/\delta Q$ varies with the cut-off radius employed however. We use cut-off radii of 3.5 ± 1 fm and carry this uncertainty through the calculation of S.

The extracted S-factors are listed in columns 6 and 7 of table 1. A comparison of columns 4, 6 and 7 illustrates the marked dependence on the Q-value of the (d, n) cross section for $l = 0$ transfer; this has an obvious practical consequence for the observation of isobaric-analogue states by the (d, n) reaction.

In column 8, we list spectroscopic factors derived from an absolute Al(d, n) measurement [5]) at $E_d = 9$ MeV using potential C in the DWBA code. Although this experiment did not resolve the high-lying levels, the high bombarding energy ensures relatively little compound nucleus formation. Our absolute spectroscopic factors agree within 30 % (i.e. the experimental error), while the relative agreement is closer, thus confirming our method of estimating the σ_{CN} in the $E_x = 1.77$ MeV group. Rubin [13]), using the photo-emulsion detection technique and $E_d = 6$ MeV, estimated $d\sigma = 31 \pm 8$ mb for the Al(d, $n_{9.33}$) reaction, which gives $(2J+1)S = 5.3 \pm 1.4$ (applying potentials B and C) in agreement with our result.

Column 10 gives absolute S-factors for the Al(d, p) reaction for the levels listed in column 9. They were derived (using potential C) from the absolute differential cross section at $\theta = 10°$ for the transitions to the ground state doublets measured [14]) at $E_d = 15$ MeV. The S-factors of the higher levels were then extracted from the relative strengths given in ref. [16]) and normalized to the ground state S-factor. The S-factors in the (d, p) and (d, n) reactions agree excellently and support our analogue state identification.

Analysis of an earlier measurement of the (d, p) reaction by Holt and Marsham [16]), however, gives S-factors five times larger. The reason for the discrepancy in S-factors derived from the two published (d, p) results is not understood.

A further point of interest, which is not obvious from columns 10 and 6 of table 1 should be mentioned. The (d, p) reaction leading to the 1.02 MeV state in ^{28}Al has a predominant $l = 2$ pattern with an estimated $l = 0$ component of only 10–15 %. However, in the (d, n) reaction to the analogue of this state, the $l = 2$ component is expected to be suppressed relative to the $l = 0$ component since at this high ex-

citation

$$\frac{\sigma_{\mathrm{DWBA}}(l_p = 0)}{\sigma_{\mathrm{DWBA}}(l_p = 2)} \approx 30.$$

Experimentally, only the $l = 0$ contribution is observed. Thus the (d, n) reaction to the analogue state serves to confirm the presence of the small $l = 0$ component in the (d, p) reaction.

The 1.37 MeV level in ^{28}Al has $J^\pi = 1^+$, and it cannot be populated by $l_n = 0$ transfer in the ordinary stripping process. Therefore the 10.669 MeV level in ^{28}Si is not the analogue of the 1.37 MeV level in ^{28}Al since the transition to the former states has an $l = 0$ component.

In addition to comparing the spectroscopic factors for the (d, n) and (d, p) reactions to individual states with $T = 1$, one can also compare the total strength $G_p(T)$ of all states excited by the transfer of a proton in the $2s_{\frac{1}{2}}$ orbital for $T = 0$ and $T = 1$ states in the final nucleus.

The strength $G_p(T)$ is defined as follows:

$$G_p(T) = \sum \frac{2J_f+1}{2J_i+1} (C_T)^2 S,$$

where J_i and J_f are the spins of the target and final nuclear states and $(C_T)^2$ an isospin statistical factor. The sum rule of French and Macfarlane [2]) gives

$$G_p(T = 0) = G_p(T = 1) = 1 \text{ for } l = 0,$$

provided the $2s_{\frac{1}{2}}$ orbitals for neutrons and protons in the target nucleus are both unfilled. (The first equality is still true even if the $2s_{\frac{1}{2}}$ orbitals are partially filled to the same extent.)

The summed strength for all transitions to $T = 0$ and $T = 1$ states is shown in the bottom of table 1. The sum for $T = 1$ states has been increased by the amount shown, which takes account of transitions to higher excited states unobserved in the (d, n) reaction. The additional strength was estimated from the corresponding (d, p) reactions. It can be seen from table 1 that the relative sum rule appears to be well confirmed, whereas the absolute summed strength G_p is too small by a factor of 2. In view of the uncertainties in absolute predictions of the DWBA code, the result does not necessarily contain information about the $2s_{\frac{1}{2}}$ occupancy in the target nucleus. This also applies to the value of G_n.

3.4. CENTROID ENERGIES OF THE *T*-STATES

The mean energies of the T-components of the $2s_{\frac{1}{2}}$ configuration may be calculated. The energies are weighted by $(2J_f+1)S$ and the $T = 1$ states include the small amount of $l = 0$ strength in higher-excited states. From these centroid energies, an average value of 3.5 ± 0.3 MeV for the isospin splitting of the $2s_{\frac{1}{2}}$ orbital is found †.

† The value is lower than that reported by Lawergren *et al.*[1]). However, at that time an estimate of the compound nucleus contribution to the (d, n) reaction had not been carried out and, in particular, the relative spectroscopic factor of the 1.77 MeV level was considerably overestimated. This had the effect of lowering the centroid energy of the $T = 0$ states and thus increasing the isospin splitting.

In the present case with little $2s_{\frac{1}{2}}$ admixture in the target ground state, this centroid splitting is approximately given by [17])

$$\Delta\bar{E} = \frac{V_1}{2A}(2T+1),$$

where T refers to the target isospin.

We obtain $V_1 = 100 \pm 10$ MeV as compared to $V_1 = 50$ MeV [ref. [17])] for splittings in ^{16}O and $92 < V_1 < 122$ MeV [ref. [18])] in Ni isotopes. In the adjacent ^{27}Al, the $T = \frac{3}{2}$ and $T = \frac{1}{2}$ centroids are separated [19]) ≈ 7.0 MeV giving $V_1 \approx 120$ MeV.

4. Conclusion

We have sought to demonstrate some of the advantages of the (d, n) reaction in the study of isobaric-analogue states, particularly with regard to $l = 0$ transitions. Such advantages compensate greatly for the experimental difficulties involved in the detection of neutrons as compared with detection of charged particles in the analogous (^{3}He, d) reaction. Extension of the work reported here to higher mass values and to higher deuteron energies will be particularly useful.

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