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STATES WITH $T = 1$ IN ^{20}Ne, ^{24}Mg AND ^{28}Si

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Abstract: The $T_z = 0$ analogues of states up to $\approx$ 2 MeV excitation in ^{20}F and ^{24}Na are located by measuring the (d, n) reactions and comparing them to the (d, p) reactions. Higher-lying analogue levels of these nuclei and of ^{28}Al are identified by relating the known (p, γ) and (d, p) reactions. Some 20 new levels have been classified as $(T, T_z) = (1, 0)$ states. The relative excitation energies of (1, 0) and (1, 1) states are strikingly similar although energy shifts of up to 100 keV occur for a few levels.

E NUCLEAR REACTIONS ^{19}F, ^{23}Na(d, n), $E_d = 2.980$ MeV; measured $\sigma(E_n, \theta)$. ^{20}Ne and ^{24}Mg deduced levels, J, π, T and spectroscopic factors.

1. Introduction

The IBIS accelerator at Harwell is well suited to the study of (d, n) reactions [1]); very high energy resolution has been achieved as a result of (i) the fast time-resolution of the time-of-flight system (< 2 ns) and (ii) the available high beam currents which, together with the large experimental area, permit long flight paths. One limitation of the system is the relatively low maximum bombarding energy ($\approx$ 3.2 MeV), which restricts the usefulness of the (d, n) reaction as a spectroscopic tool. However, in our experience stripping analysis is practicable (and accurate l- and S-values can be extracted) provided the cross section for stripping is reasonably high as compared to the compound nucleus (CN) cross section. The properties of the direct reaction (DI) mechanism are such that, within the same nucleus, the proton transfer probability is much higher for transitions with low Q-values than for those with high Q-values. As a result, the CN cross section is comparatively small (< 10 % of the DI cross section) for transitions to high-lying levels, i.e. the region where isobaric analogue states are to be found. Low-lying levels, on the other hand, are more difficult to treat. In that case one has to subtract the CN cross section whose magnitude is indicated by the fluctuations observed in a yield function measured using a thin target. A more quantitative treatment is to measure the angular distribution of the (d, n) reaction averaged over an energy range large enough for interference terms between direct interaction and compound nuclear amplitudes to average to zero. One may then

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write, using an obvious notation

$$\left(\frac{d\sigma}{d\Omega}\right)_{obs} = \left(\frac{d\sigma}{d\Omega}\right)_{CN} + \left(\frac{d\sigma}{d\Omega}\right)_{DI}.$$

The compound nucleus term may be estimated using the Hauser-Feshbach formalism giving the direct component by subtraction.

Since states with $T = 1$ in even nuclei are analogues of low-lying states, they are expected on the average to have larger single-particle components than the states with $T = 0$ which form the background at the same excitation energies. The level density at this energy ($\approx$ 10 MeV) is still fairly low, and one anticipates little spreading of the particle strength belonging to the $T = 1$ states. The analogue states will therefore be comparatively strongly populated in the (d, n) reaction and thus identified.

We chose to study the ^{19}F(d, n)^{20}Ne, the ^{23}Na(d, n)^{24}Mg and the ^{27}Al(d, n)^{28}Si reactions since the high Q-values allow the detection of many high-lying states with $T = 1$ in each final nucleus and since large components of $l = 0$ stripping are expected.

In this paper we wish to report the results of the two former reactions. The analysis follows that of the ^{27}Al(d, n)^{28}Si reaction which has already been reported [1], and the reader is referred to that paper for details. Suffice it here to give the criteria on which we base the $T = 1$ identification in a $T_z = 0$ nucleus.

(i) The l- and S-values † extracted from the (d, n) reaction should be identical to those extracted from the (d, p) reaction. Unfortunately, unique J-values cannot be measured and compared in either reaction.

(ii) The sum over all $l = 0$ transitions

$$G = \sum S(2J_f+1)(2J_i+1)^{-1})\, C^2$$

should be equal to unity in both the $T = 0$ and the $T = 1$ states provided that $1s_{\frac{1}{2}}$ shell is empty.

In sect. 4, we shall review unbound levels and replace the information obtained in the (d, n) reaction with that obtained in the (p, γ) process. The method of analysis of (p, γ) reactions using the statistical tensor techniques [3]) often determines J^π and l-values and from Γ_p one can determine the partial reduced width γ^2 which may be compared to the S-factor determined in the (d, p) reaction.

The (d, n) measurements were performed at a bombarding energy of 2.980 ± 0.005 MeV with a neutron flight path of 6 m. The targets consisted of LiF and NaF deposits $\approx 150\ \mu g/cm^2$ thick on Ta backings. Only *relative* cross sections were measured since, for nuclei as light as mass 20 and 24, the DWBA theory gives large uncertainties in absolute S-factors although giving their relative values with some precision.

It should also be remembered from ref. [2]) that even the *relative* S-factors we extract are, to some extent, a function of the parameters employed in the DWBA programme. The most serious uncertainty so introduced is due to the employment of a so-called cut-off radius. It appears to be necessary to use this device to correct for

† In the isospin notation where the coupling factor C^2 is included in the analysis.

the incomplete description of the incoming deuteron wave and make the experimental and DWBA predicted angular distributions agree when $l \neq 0$. The artificial cut-off parameter most drastically effects the high-Q transitions. The S-factors of the $T = 1$ states are thus little effected but, unfortunately, the application of eq. (1) becomes more affirmative than decisive.

2. The ^{19}F(d, n)^{20}Ne reaction

The lowest state with $T = 1$ is believed [4]) to lie at 10.270±0.009 MeV excitation in ^{20}Ne. The Q-value of the ^{19}F(d, n) reaction is 10.646 MeV, and the analogues of states below 2 MeV excitation in ^{20}F are thus energetically accessible. From the

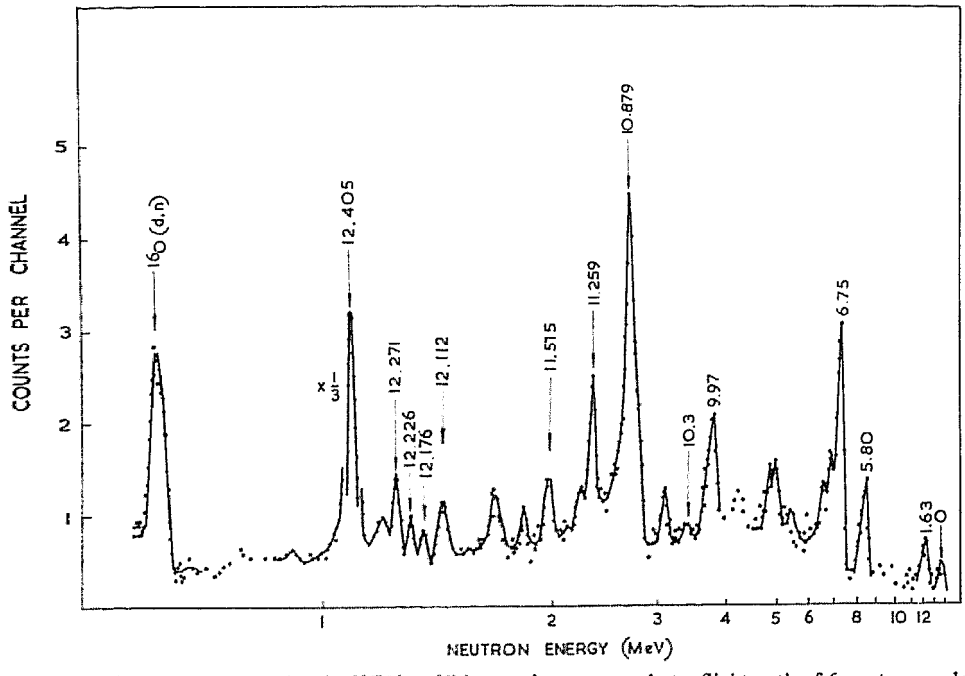

Fig. 1. Neutron spectrum for the ^{19}F(d, n)^{20}Ne reaction measured at a flight path of 6 m at an angle of 0°.

^{19}F(d, p)^{20}F reaction, we know [5]) that some of these states have large s- and d-wave single-particle components, i.e. the states are thus expected to be well populated in the (d, n) reaction.

The neutron spectrum at $\theta = 0°$ is given in fig. 1. A number of narrow neutron groups leading to levels in ^{20}Ne are superimposed on a background of continuously distributed neutrons probably emerging from three body decays, e.g.

$$^{19}\text{F} + \text{d} \rightarrow {}^{16}\text{O} + \alpha + \text{n}; \; Q = +5.9 \text{ MeV}.$$

The sharp group leading to a level at 10.879 MeV in ^{20}Ne is only resolved from the

neutron group corresponding to the $^{12}C(d, n_0)$ reaction at angles larger than 40°. Observed angular distributions are compared with DWBA predictions of the most probable l-value in fig. 2. Transitions to the states of interest are summarized in table 1. In this table we introduce the notation E_1^x for the excitation of a level in the $(T, T_z) = (1, 1)$ nucleus and E_0^x for the energy difference between the excitation of an analogue level and the analogue of the ground state in the (1, 0) nucleus. The spec-

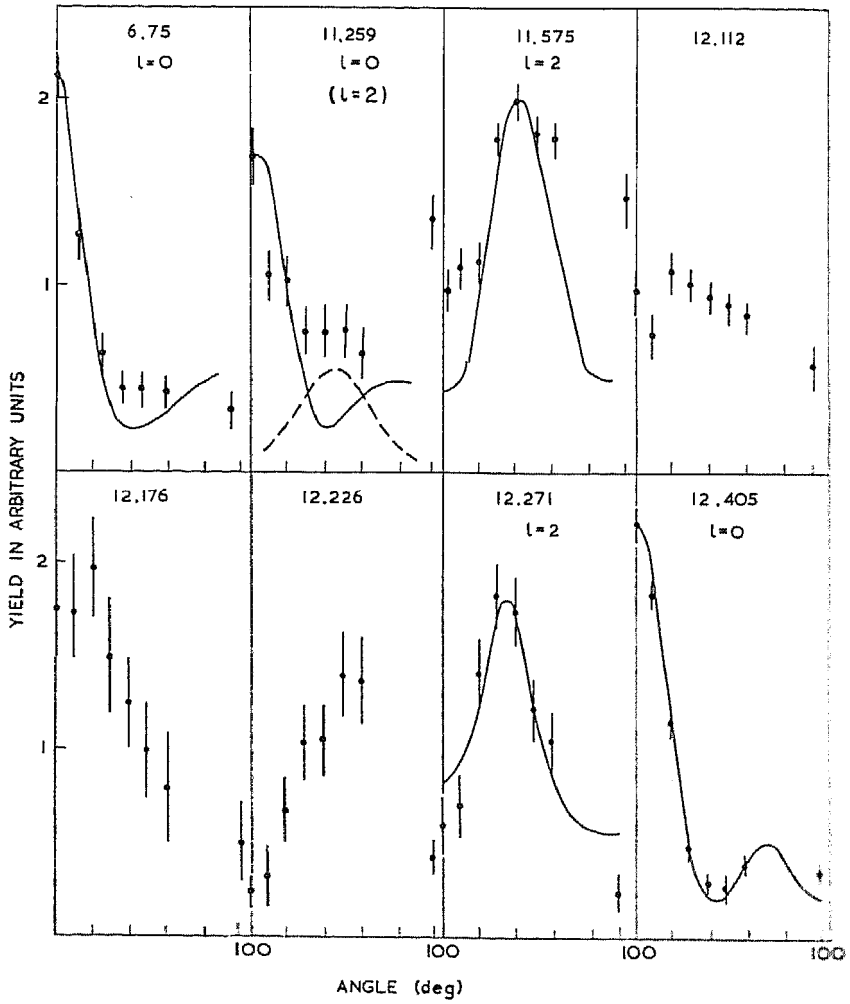

Fig. 2. Angular distributions for some of the prominent neutron groups in the $^{19}F(d, n)^{20}Ne$ reaction. The solid curves are calculated DWBA distributions for l-values shown.

troscopic factors were extracted in the same manner as in ref. [2]), using the same optical parameters to describe the incoming deuteron wave. This method gives relative spectroscopic factors to an accuracy of $\pm 50\,\%$ for the $l = 0$ states with which we are concerned here.

The (d, p) reaction [5]) demonstrates that a few of the low-lying ^{20}F levels are only weakly populated by the stripping process with no distinctive stripping patterns. The identification of the analogues of those levels is less certain than of states with large single-particle components. However the two fairly intense (d, n) transitions with $l = 2$ leading to the $T_z = 0$ analogues of the 0.65 and the 2.05 MeV levels in ^{20}F are quite unmistakable since they are the strongest $l = 2$ seen, and the ratio of their strengths is very close to that of the strong (d, p) transitions to the two $T_z = 1$ states. The identification in the (d, n) reaction of the $T_z = 0$ analogues of the levels in ^{20}F around 1 MeV excitation is somewhat less certain. Such transitions are weak and, in addition, (d, p) reaction studies [5]) differ in their assignment of l-values and strengths to these levels. However, the $l = 0$ component seen in the (d, n) reaction to the level(s) at 11.259 MeV is consistent with its interpretation as the analogue of

TABLE 1

States identified in ^{19}F(d, n) and ^{19}F(d, p) reactions

^{20}Ne* (MeV)	E_p^x(MeV) ($E-10.27$)	T	l_p	$S_p(2J_f+1)C^2$ (relative)	$^{20}F^*$ E_n^x(MeV)	l_n	$S_n(2J_f+1)$ [a]) (relative)
0 ±0.5		0	(0)	(0.6) [b])			
6.75 ±0.10		0	0	0.7			
10.3 ±0.1	0	1			0.00		
10.879±0.010	0.61	1	2	2 ±1	0.65	2	2
11.259±0.010	0.99	(1)	0 (2)	0.10±0.05 0.1 ±0.1	0.99 1.06	(2) (0)	0.2 small
11.575±0.010	1.31	(1)	2	0.6 ±0.2	1.31	(2)	0.2
12.112±0.010	1.84	(1)	small; forward peaked		1.85	small; forward peaked	
12.176±0.010	1.91	(0)	small; forward peaked				
12.226±0.010	1.96	(1)	small; backward peaked		1.97	small; backward peaked	
12.271±0.010	2.00	1	2	1.8 ±0.6	2.05	2	2
12.405±0.010	2.14	0	0	0.3 ±0.2			

[a]) Ref. [5]).
[b]) Extracted from ref. [7]) (see text).

the 1.06 MeV level in ^{20}F which has a spin of 1^+ from a β-decay experiment [6]). The analogue of the 0.83 MeV level in ^{20}F will be only weakly excited compared to the analogue of the 0.65 MeV level and is not resolved in the (d, n) experiment. The intensely populated state at 12.405 MeV excitation has definite $T = 0$ character since (i) it lacks an analogue state in ^{20}F and (ii) has a considerable alpha-particle width measured in the ^{16}O(α, γ)^{20}Ne reaction [4]). On the other hand, the measured states with $T = 1$ classification have small Γ_α as required.

From the results of (d, p) reaction studies, one may estimate the size of the $s_{\frac{1}{2}}$ component in ^{20}F states above the energetically accessible energy region, i.e. above the 1.31 MeV state. In fact, only about 10 % of the total strength is detected in the present experiment. This means that eq. (1) cannot be tested with any accuracy in this reaction. However, for future reference we evaluate the S-factors for $l = 0$ transitions

detected in the present reaction study. This is easy for the levels above 6.75 MeV where the fraction of the cross section due to CN processes is low. On the other hand, for the ground state transition CN is so large that an S-factor cannot be reliably obtained from our measurements. For our analysis we have used a value taken from a published [7]) measurement of the $^{19}F(^3He, d)$ reaction at $E_{He} = 9$ MeV, normalizing to the S-factor of the 6.75 MeV state.

3. The $^{23}Na(d, n)^{24}Mg$ reaction

Rickey *et al.* [8]) have used the fact that β-decays from a $T = 1$ parent to the lowest $T = 1$ states of a daughter nucleus are superallowed to show that the lowest $T = 1$

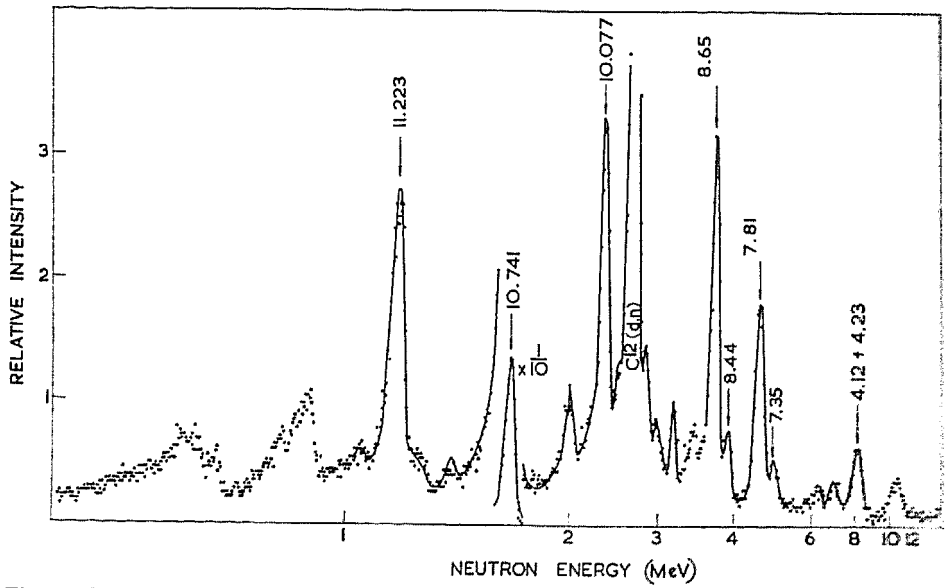

Fig. 3. Neutron spectrum for the $^{23}Na(d, n)^{24}Mg$ reaction measured at a flight path of 6 m at an angle of 0°.

level in ^{24}Mg is at an excitation of 9.512 ± 0.009 MeV. This agrees well with Wilkinson's [9]) prediction of $9.504 + 0.012$ MeV. The ground state and the first excited state of ^{24}Na are known to be weakly populated in the (d, p) reaction and so the 9.512 MeV level is not expected to be particularly prominent in the present reaction.

The spectrum at $\theta = 0°$ is shown in fig. 3 and the angular distributions of transitions with $l = 0$ are illustrated in fig. 4 together with the DWBA fit. We identify $l = 0$ transitions leading to levels at 7.81 and 8.65 MeV instead of at 7.4 and 8.4 MeV as indicated by earlier measurements [10]); otherwise our level assignment agrees with the previous low resolution measurement. The S-factor of the lowest $l = 0$ component is estimated from ref. [10]) normalizing to the S-factors of the levels at 7.81 and 8.65 MeV. All observed $T = 0$ groups with $l = 0$ and all identified $T = 1$ states are given in table 2.

Three strong proton groups populate states at 0.56, 1.35 and 1.84 MeV in the ^{23}Na(d, p)^{24}Na reaction [11]), and we find similarly three corresponding strong neutron groups to states between 10 and 11 MeV excitation in ^{24}Mg. The relative S-factors of these levels agree excellently and so there is a high degree of confidence in the

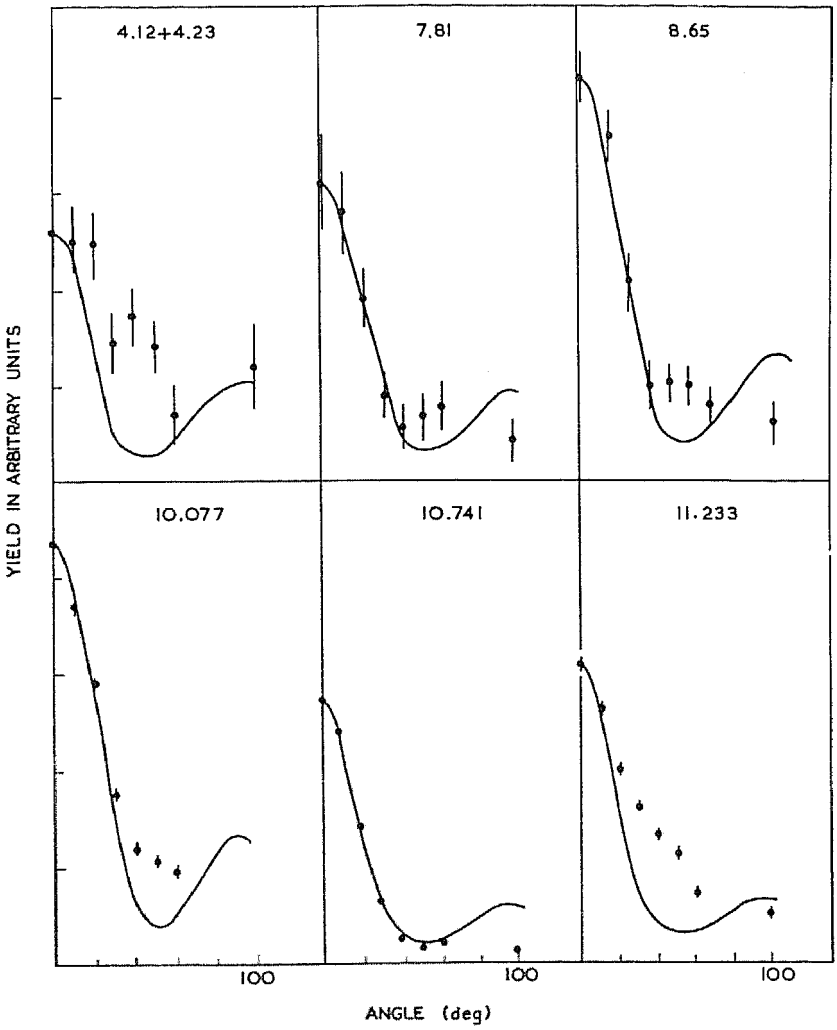

Fig. 4. Angular distributions for some of the prominent neutron groups in the ^{23}Na(d, n)^{24}Mg reaction. The solid curves are calculated DWBA distributions for $l = 0$ transitions.

analogue identification. Weaker groups are seen which probably correspond to the ground and first excited state of ^{24}Na. The $l = 0$ group to the analogue of the 1.84 MeV level appears wider than well resolved groups on either side and presumably contains an unresolved component which could correspond to the 1.88 MeV level in

^{24}Na. In this case, since we are able to measure the larger part of the total $l = 0$ component in both isospin states, eq. (1) can be invoked in order to further check the T-identification. It gives $G = 1.8 \pm 0.6$ in the $T = 1$ state if we normalize $G = 1.0$ for the $T = 0$ component.

TABLE 2

States identified in the ^{23}Na(d, n) and ^{23}Na(d, p) reactions

^{24}Mg* (MeV)	E_0^x(MeV) (E−9.513)	T	l_p	$S_p(2J_f+1)C^2$ [a] (relative)	^{24}Na* E_1^x(MeV)	l_n	$S_n(2J_f+1)$ [b] (relative)
4.23 ±0.30		0	0	(0.5) [c]			
7.35 ±0.10		0	0	0.2			
7.81 ±0.10		0	0	1.2			
8.65 ±0.05		0	0	1.0			
9.524±0.050	0.012	(1)	(2)	3.5±3.0	0.00	2	3.3
9.995±0.010	0.483	(1)	(2)	2.5±2.0	0.473	2	1.9
10.077±0.010	0.564	1	0	0.5	0.564	0	0.5
10.741±0.010	1.230	1	0	3.0	1.341	0	3.0
11.223±0.010	1.710	1	0	0.9	1.844	0	1.0

[a]) 30 % errors.
[b]) Ref. [11]).
[c]) Extracted from ref. [16]).

4. Analogue state identification using published data on (p, γ) reactions

Thus far we have located a number of analogue levels corresponding to $T_z = 1$ levels below ≈ 2 MeV excitation in the doubly odd nuclei ^{20}F, ^{24}Na and ^{28}Al. We have gained some knowledge of the extent of the agreement between the $(T, T_z) = (1, 0)$ and the (1, 1) pairs. The following trend is observed:

(i) The energy difference $|E_0^x - E_1^x|$ between corresponding states may be 100 keV, but in the majority of cases it is less than 50 keV.

(ii) The relative S-factors for the series of $T_z = 1$ states and the corresponding series of $T_z = 0$ states agree within experimental uncertainties, i.e. ≈ 30 %.

The (p, γ) reaction provides much the same information for unbound states as does the (d, n) reaction for bound states, and we can use it to locate higher-lying analogue states guided by the rules above. In fact, the resonant capture process determines the total angular momentum J and the proton width; the parity can often be inferred from the de-excitation mode. Since the analogue states should have negligible α-particle width and, with no other particle channels open, the total width should be approximately equal to the proton width. However, there are in general no J-values available from (d, p) reactions to be compared with, and the exact value of r_0 in eq. (3) below is subject to some uncertainty. The reduced width can be extracted from the resonance width through the relation [12])

$$\gamma_p^2 = \frac{A_l^2}{2kr_0}\Gamma_p, \qquad (2)$$

where k is the wave number, r_0 the interaction radius and A_l the penetration factor.

The reduced width of these analogue states may likewise be deduced [13]) from the S-factors obtained in the (d, p) reaction

$$S_n = \theta_0^{-2}\,\gamma_n^2\,\frac{2A}{3(A+1)}\,\frac{Mr_0^2}{h^2}, \tag{3}$$

where θ_0^2 is the reduced single-particle width [13]) and M the nucleon mass. The reduced widths for analogue levels γ_p and γ_n, respectively, should be identical.

Resonances in the $^{19}F(p,\gamma)^{20}Ne$ reaction [14]), the $^{23}Na(p,\gamma)^{24}Mg$ reaction [15]) and the $^{27}Al(p,\gamma)^{28}Si$ reaction [16]) have been extensively studied below $E_p \approx 1.5$ MeV, and the gamma-ray angular correlation measurements have been analysed in terms of the statistical tensors [3]) of the populated resonance level. A summary of the most convincing identifications are given in table 3. The absolute reduced widths (γ^2) only agree moderately well, but this probably reflects the unreliable values

TABLE 3

States with $T = 1$ identified in (p, γ) and (d, p) reactions

$T_z = 0$ Nucleus	E_{res} (MeV)	E^x (MeV)	E_0^x (MeV)	l_p	$J\pi$	γ_p^2 (keV)	$T_z = 1$ nucleus	E_1^x (MeV)	l_n	γ_n^{2*} (keV)	Confidence in T-assignment
^{20}Ne	0.669	13.509	3.24	0	1^-	300	^{20}F	3.49	0	15	A
^{20}Ne	1.422	14.224	3.95	0	1^-	25	^{20}F	4.08	0	1.5	B
^{24}Mg	0.676	12.340	2.816	2	3^-	<1000	^{24}Na	2.98	2	100	B
^{24}Mg	0.871	12.528	3.004	0	1^-	250	^{24}Na	3.409	0	300	B
^{24}Mg	1.019	12.669	3.145	1	2^-	350	^{24}Na	3.37	1	125	B
^{24}Mg	1.172	12.816	3.292	0	1^-	25	^{24}Na	3.582	0	100	B
^{28}Si	0.992	12.536	3.201	0	3^-	2.5	^{28}Al	3.347	0	1.6	A
^{28}Si	1.117	12.658	3.323	1	4^-	50	^{28}Al	3.461	1	20	A
^{28}Si	1.198	12.736	3.401	1	3^-	300	^{28}Al	3.591	1	50	A

of reduced widths extracted from the (d, p) reaction using the "plane wave Born approximation" of refs. [5,11]); the relative widths agree better however. Still, some of the identifications appear more substantiated than others and a rating, A and B respectively, has been introduced in table 3 accordingly. For all the quoted levels the α-particle widths are small. The surrounding $T = 0$ states have larger Γ_α on the other hand.

Jaffe and Harchol [17]) have also examined the known resonances in the $^{27}Al(p,\gamma)^{28}Si$ reaction in search for possible $T = 1$ components. We disagree about the identity of the 1.198 MeV resonance. Whatever way this is viewed, inconsistencies in the reduced widths will remain, but the proposal in table 3 gives seemingly the best agreement between the (1, 1) and the (1, 0) states. Still, the identifications should be treated with some caution. In fact, only the states marked A are fairly certain $T = 1$ states.

The 26 analogue levels identified in this paper and in ref. [2]) are presented diagrammatically in fig. 5. The excitation energies E_1^x and E_0^x have been plotted for masses 20, 24 and 28.

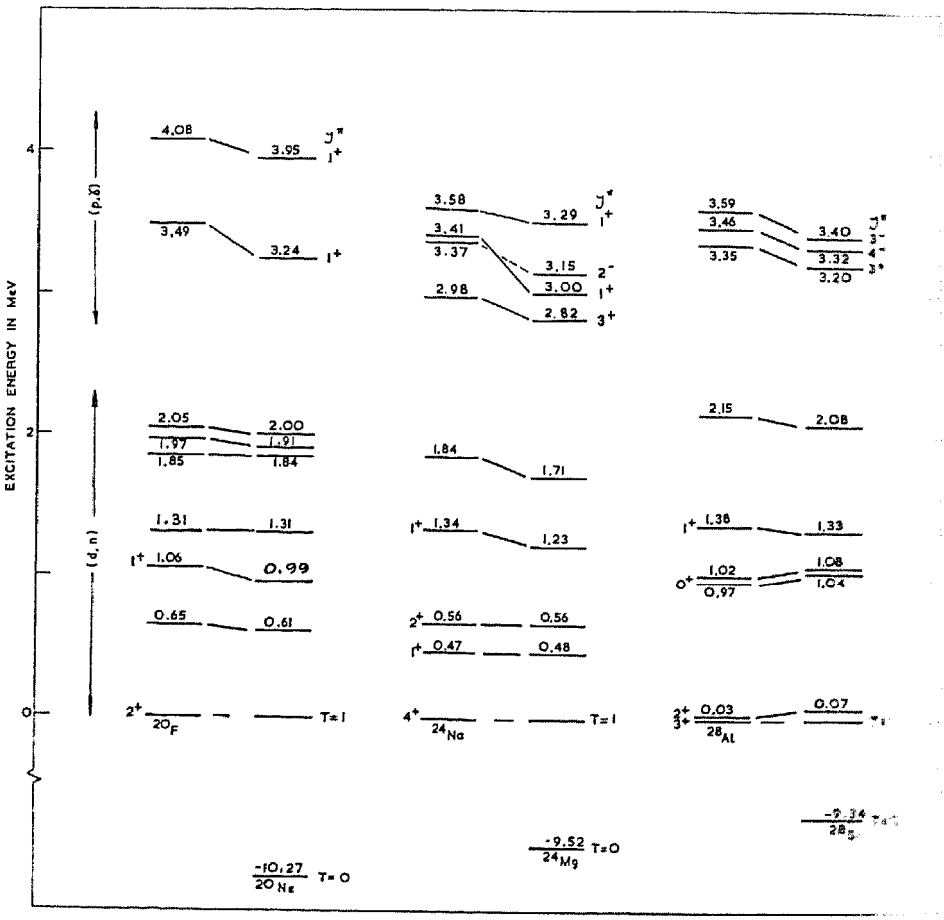

Fig. 5. The excitation of states with $T = 1$ in nuclei having $T_z = 1$ and $T_z = 0$ isospin projections.

5. The energy splitting of the mean $T = 1$ and $T = 0$ components of the $s_{\frac{1}{2}}$ orbit

The mean excitation of the two T-components in each nucleus is determined from the observed level energies weighted by the measured S-factors. These values are given in table 4 together with those [2]) for ^{28}Si.

The difference of the average energies in the T-states is theoretically accounted for by the T-dependent part of the average central potential [18]) in which the $s_{\frac{1}{2}}$ nucleon moves, i.e. $\Delta E = V_1(2T+1)/2A$ where V_1 is the depth of the potential. The deduced values of V_1 are also given in table 4.

6. Discussion of level energies

It has thus proved possible to identify most analogue states that are energetically accessible in the present study. States in the (1, 1) system which were not observed in the (1, 0) system via the (d, n) reaction, were only weakly populated in the (d, p) reaction.

The close correspondence between the energies of the states in the (1, 1) and the (1, 0) systems is impressive, as is the good resemblance between spectroscopic factors observed in the stripping reactions. At higher energies, where (p, γ) reaction was employed, there are some deviations from the predicted reduced nucleon widths. A possible explanation may be sought in isospin impurities which tend to increase with increasing excitation energy. Through this effect, Γ_α may be large although the actual admixture is relatively small and our assumption, $\Gamma_p = \Gamma_{tot}$ is erroneous.

TABLE 4

Centroid energies (in MeV) for the $s_{\frac{1}{2}}$ orbit

Nucleus	$E(T=0)$	$E(T=1)$	ΔE	V_1
^{20}Ne	5.5	13.6	8.1	160±100
^{24}Mg	7.5	11.1	3.6	80± 30
^{28}Si	6.5	10.0	3.5	100± 10

In order to compare the observed level positions with theoretical predictions, one needs to know the configurations of the (1, 1) states. It is clear, however, that the configurations are strongly mixed and that collective excitations are present in these doubly-odd nuclei. The rotational model has had some success in adjacent odd and even nuclei. For doubly-odd nuclei, one must expect strong K-mixings which blurr the distinction between particle and collective excitations. A realistic treatment of the observed systematics must take this complicated structure into account, and more detailed discussion left till that occasion.

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