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Time Sliced Three Dimensional Inverse Image Reconstruction of Objects in Highly Scattering Media

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ABSTRACT

An inverse image reconstruction approach that makes use of an algorithm based on the diffusion approximation of the radiative transport theory and a sequence of picosecond-duration slices of transmitted two-dimensional (2D) light intensity distribution for fast 3D image reconstruction is presented. The results of simulation and experiment for a cylindrical geometry are presented. Object localization in the lateral dimensions is better than that in the axial direction. The observed difference in axial and lateral resolutions are analyzed by introducing the concept of "longitudinal sensitivity".

Keywords: Inverse reconstruction, time-sliced imaging, transillumination

1. INTRODUCTION

In the article, we present an inverse image reconstruction (IIR) approach that makes use of a sequence of picosecond-duration slices of transmitted two-dimensional (2D) light intensity distribution measured at one plane, instead of surrounding the medium, for fast 3D image reconstruction. This approach may allow for simpler 3D imaging of breast.

2. OPTICAL IMAGING SYSTEM

The experimental system, described in detail elsewhere,[1] makes use of $800\,nm$, $150\,fs$, $1\,kHz$ repetition-rate pulses from a Ti:sapphire laser for sample illumination and an electronic gated image intensifier coupled to a CCD camera to record time-sliced images. The scattering medium was a suspension of Intralipid-10% (Kabi Pharmacia Inc., Clayton, North Carolina) in water, adjusted to provide an estimated reduced scattering coefficient $\mu_s' = 0.4\,mm^{-1}$ and an absorption coefficient $\mu_a = 0.02\,mm^{-1}$ at wavelength $800\,nm$. The Intralipid suspension was held in a $60\,mm$ long and $200\,mm$ diameter cylindrical Plexiglas cell. The object was a $3 \times 3 \times 10\,mm^3$ rectangular parallelepiped made of aluminum and painted black. It was suspended on axis at a distance of $15\,mm$ ($z = 45\,mm$) from the exit plane. The gate width was adjusted to $80\,ps$ and time-sliced transmitted intensity distribution, $I(x, y, t_i)$, were recorded over a $5\,ns$ range by varying the gate position in steps of $100\,ps$.

3. ALGORITHM

The inverse image reconstruction algorithm is based on the diffusion approximation of the radiative transport theory for photon migration in a scattering medium. For the linear inversion case, the forward problem can be written in matrix form:

$$Y(\rho_d, \phi_d, z_d, t) = \sum_{\rho,\phi,z} W(\rho_d, \phi_d - \phi, z_d, t|\rho, 0, z)X(\rho, \phi, z) \quad (1)$$

where $Y = -\log(I/I_0)$, I and I_0 are intensities with and without the inhomogeneity respectively, $X = \Delta\mu_a c$ or $\Delta D c$ depends on the type of the inhomogeneity, and W is the weight function. The convolution over ϕ in Eq. (1) can be eliminated by a Fourier transform in the $\hat{\phi}$ direction:

$$Y_k(\rho_d, z_d, t) = \sum_{\rho,z} W_k(\rho_d, z_d, t|\rho, z)X_k(\rho, z) \quad (2)$$

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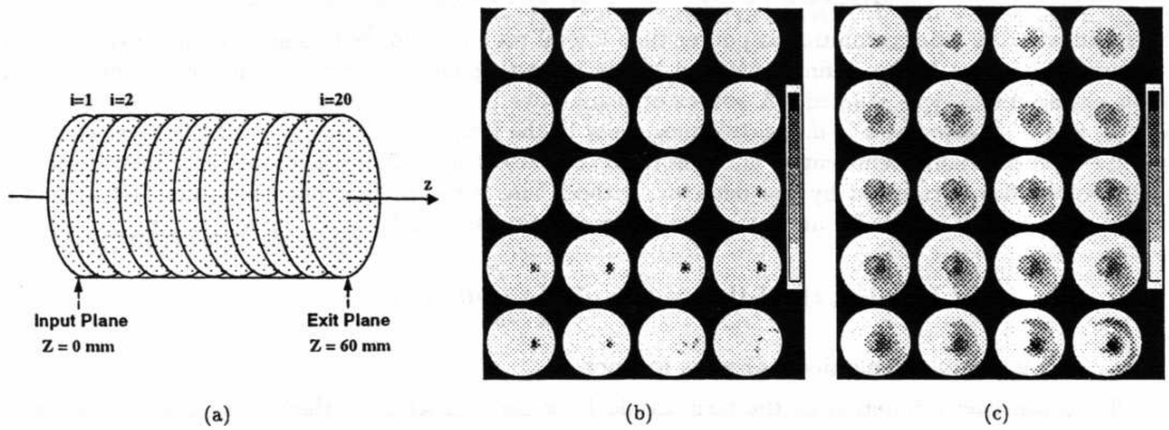

Figure 1. (a) The cylindrical slab is uniformly divided into 20 slices along the z-axis. (b) Images reconstructed from simulated data when the object was located at $z = 45\,mm$. (c) Images reconstructed from experimental data with the object located at $z = 45\,mm$. The sequence of circles represents images at $3\,mm$ intervals along the cylinder axis.

where Y_k, W_k and X_k are Fourier transforms of Y, W and X over ϕ respectively. The inverse problem is solved using a Tikhonov regularization method[2] with a diagonal form of regularization matrix $\lambda(k)$. The suitable regularization parameters $\lambda(k)$ are chosen by the L-curve criterion.[3] In our calculation, the sample cell is divided into $N_r \times N_\phi \times N_z = 10 \times 36 \times 20$ voxels. One inversion can be completed in 1 minute in a Silicon Graphics Origin200 workstation when the weight matrix is pre-computed.

4. RESULTS

4.1. Three-dimensional image from simulated data

In the simulation, we assumed a cylindrical slab of thickness $z_0 = 60\,mm$, and a disk detector of radius $R_d = 30\,mm$. The thickness of the cylindrical slab and its angle 2π was uniformly sliced into 20 and 36 parts respectively, while its radius R_d was divided into 10 parts scaled such that each voxel has equal-volume (Figure 1(a)). Sampled data both with and without hidden objects were used to obtain $Y = -\log I/I_0$ after added Gaussian distributed noise of $\pm 5\%$. The background was assumed to have a reduced scattering coefficient $\mu_s^{0\prime} = 0.4\,mm^{-1}$, and an absorption coefficient $\mu_a^0 = 0.02\,mm^{-1}$. The optical parameters of the inhomogeneity were $\mu_s' = \mu_s^{0\prime}$ and $\mu_a = 0.4\,mm^{-1}$, which was put in a voxel at the $15th$ division along z-axis.

The reconstructed image of $\Delta\mu_a$ is shown in Figure 1(b). The 20 consecutive circles in the figure represent the images at different position along the z-axis from input plane ($z = 0$) to exit plane ($z = 60\,mm$). A gray scale is used to denote the relative value of the change of the absorption coefficient. The reconstructed image locates the object in the neighborhood of where it was placed. The object appears to spread out both in the lateral and axial directions, a consequence of the scattering and the diffusion approximation used for reconstruction. The image spreads out to approximately 6-7 mm in the lateral, (ρ, ϕ) plane, and to approximately 10-15 mm along the axial, z-direction.

4.2. Three-dimensional image from experimental data

In the inverse image reconstruction using experimental data, we approximated the laser beam to be a point source located at the center of the $z = 0$ plane. The ρ boundary of the container was assumed to be infinity.

Figure 1(c) shows a 3D image of the object as a sequence of frames at $3\,mm$ intervals along the axial direction from input plane to exit plane reconstructed using experimental data. The lateral position of the object is reconstructed to be near the cylinder axis in the image map, as the actual object was placed. However, the image is spread out over approximately $20\,mm$ along the axial direction and peaked more towards the detector end than the actual object position. The worse result from experimental data indicates higher noise in experiment than 5% and existence of deviation between theoretical model and experiment.

5. LONGITUDINAL SENSITIVITY ANALYSIS

To understand why the axial resolution is poorer than lateral resolution for both simulated and experimental data, a one-dimensional model for an infinite uniform highly scattering medium with a diffusion coefficient D and an absorption coefficient μ_a was studied. A source of plane-wave pulse with intensity $S\delta(z+d)\delta(t)$ was located at $z=-d<0$, and a detector at $z=d$. An inhomogeneity in the form of an absorptive infinite layer of infinitesimal thickness Δz with diffusion coefficient D and absorption coefficient $\mu_a+\Delta\mu_a$ was placed at z where $-\infty<z<\infty$. The time-resolved fluxes received by the detector, without and with the inhomogeneity, are $I_0(t,z)$ and $I_1(t,z)$, respectively. The change of flux for an absorptive inhomogeneity is given by:[4]

$$\Delta I(t,z) = I_0(t,z) - I_1(t,z) = S\Delta\mu_a\Delta z\left\{c\int_0^t d\tau G(d,z,t-\tau)G(z,-d,\tau)\right\}, \tag{3}$$

where $G(z_1,z_2,t)$ is the one-dimensional Green's function.

We define a sensitivity function as the term inside {} of Eq. (3) which is the change of flux measured by the detector at time t when an absorption layer of unit thickness and of unit change of absorption coefficient is placed at position z for an incident pulse of unit intensity. The sensitivity function is found to be[5]

$$W_{\text{inf}}(t,z) = \frac{\exp(-\mu_a ct)}{4D}\,\text{erfc}(\alpha\max(|z|,d)) \tag{4}$$

where $\alpha^{-1}=\sqrt{Dct}$ is the characteristic length. $W_{\text{inf}}(t,z)$ is totally independent of z when the absorptive layer is between the detector and the source. Therefore, for one-dimensional case, reconstruction using only transmission measurements is not sensitive to axial position of the inhomogeneity. In a 3D transmission case, such as that in section 3, it is not so severe but limits the longitudinal resolution, as we have observed in section 4.

6. CONCLUSION

An inverse image reconstruction approach that uses a sequence of time slices of two-dimensional light intensity distribution, and a fast algorithm based on the diffusion approximation of the radiative transport theory is presented. Results of reconstruction with simulated and experimental data are also presented. The image spreads to about 6-7 mm in the lateral, and to approximately 10-15 mm along the axial direction for one voxel inhomogeneity in simulation while the image spreads out to approximately 20 mm along the axial direction for experimental data. The difference in lateral and axial resolutions may be explained by the concept of "longitudinal sensitivity".

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